Family Letter

Content Overview

Dear Family,

Your child is learning math in an innovative program called *Math Expressions*. The program emphasizes understanding and fluency. Your child will

- connect math to daily life.

- solve a problem by understanding and representing the situation and then finding the solution.

- work with and explain strategies to other students.

Visualizing and talking about math is very important. This approach helps children feel that they can learn math. They do learn math, enjoy doing so, and feel good about themselves as math learners.

Recent research shows that effort is important in getting smarter and in doing well in math. This is called effort-produced ability. Students who work hard, think and participate in class, and do their homework do learn math. So encourage your child to work hard and to know that he or she can learn math.

Your child will have math homework almost every day. Homework is important for math learning. Make a special time for homework in a quiet place. Ask your child to explain to you some problem he or she did.

To make concepts clearer, the *Math Expressions* program uses some special methods and visual supports. Some of these for Unit 1 are shown on the back. More information for families is on the Web at http://www.thinkcentral.com/.

If your child has problems with math, please send me a note or talk to me to see how we can work together to help your child.

Thank you. You are vital to your child's learning.

Family Letter

Content Overview

In our math class, we are exploring the ideas of rate, ratio, and proportion.

A rate tells how much is used repeatedly in a given situation. For example, $8 *per package* means $8 *for each package* or $8 *for every package*.

The ratio of one number to another is a simple way to express the relative size of two quantities or measurements.

A proportion is an equation that shows two equivalent ratios. It can be written 6:14 = 15:35   or   6:14 :: 15:35.

In a proportion problem, one of the four numbers is unknown. For example:

> Grandma made applesauce using the same number of bags of red apples and bags of yellow apples. Her red apples cost $6 and her yellow apples cost $14. I used her recipe but made more applesauce. I paid $35 for my yellow apples. How much did my red apples cost?

Factor Puzzle

Red Yellow

	3	7	
2	6	14	2
5	15	35	5
	3	7	

The problem makes this proportion:

$$6:14 = c:35$$

To solve this proportion, we can put the ratios in a Factor Puzzle.

$$c \text{ is } 3 \times 5 = 15$$

The Factor Puzzle is from the rows of the ratio table that are × 2 (• 2) and × 5 (• 5) of the basic ratio 3:7. Factor Puzzles enable your child to understand and solve challenging proportion problems.

Discuss with your child how you use proportions in your life, such as when you double a recipe.

If you have any questions, please call or write to me.

Sincerely,
Your child's teacher

Ratio Table

	R	:	Y
Bags	3	:	7
1	3	:	7
2	6	:	14
3	9	:	21
4	12	:	28
5	15	:	35
6	18	:	42
7	21	:	49
8	24	:	56
9	27	:	63

• 2

• 5

Estimada familia,

Su hijo está aprendiendo matemáticas mediante un innovador programa llamado *Expresiones en matemáticas*. Este programa enfatiza la comprensión y el dominio de los conocimientos. Con este programa, su hijo aprenderá a:

• relacionar las matemáticas con la vida diaria.

• resolver problemas mediante la comprensión y la representación de diferentes situaciones.

• trabajar con diferentes estrategias y a explicarlas a otros estudiantes.

La visualización y las charlas matemáticas son muy importantes. Cuando se usan, facilitan el aprendizaje. Mediante ellas el alumno aprende, disfruta mientras lo hace, y adquiere una buena autoestima como estudiante de matemáticas.

Estudios recientes han demostrado que esforzarse es muy importante para el desarrollo de la inteligencia y el estudio de las matemáticas. A este proceso se le llama adquisición de destrezas mediante esfuerzo. Para aprender, los estudiantes deben trabajar, concentrarse y participar durante la clase, y realizar tareas en la casa. Por esto, le pedimos que anime a su hijo a completar sus trabajos.

Casi diariamente su hijo tendrá tarea para la casa. La tarea es una parte muy importante del aprendizaje de las matemáticas. Para realizarla, designe un tiempo especial en un lugar tranquilo. Al terminar, pida a su hijo que le explique cómo resolvió algunos de los problemas.

Para lograr que los conceptos sean más claros, el programa *Expresiones en matemáticas* utiliza algunos métodos especiales y refuerzos visuales. Algunos de los que corresponden a la Unidad 1 se muestran en la parte de atrás. Puede hallar más información para la familia en Internet en http://www.thinkcentral.com/.

Si su hijo tiene problemas en la asignatura de matemáticas, por favor comuníquese conmigo para que unidos hallemos la mejor manera de ayudarlo.

Gracias. Usted es imprescindible para el aprendizaje de su hijo.

En nuestra clase de matemáticas, estamos estudiando los conceptos de tasa, razón y proporción.

Una tasa indica cuánto se usa algo repetidamente en una situación dada. Por ejemplo, $8 *por paquete* significa $8 *por cada paquete*.

La razón de un número a otro es una manera simple de expresar el tamaño relativo de dos cantidades o medidas.

Una proporción es una ecuación que muestra dos razones equivalentes. Puede escribirse así:

6:14 = 15:35   o   6:14 :: 15:35.

En un problema de proporción, uno de los cuatro números es un número desconocido. Por ejemplo:

Para hacer puré de manzanas, mi abuelita usó el mismo número de bolsas de manzanas rojas que de manzanas amarillas. Las bolsas de manzanas rojas costaron $6 y las de amarillas $14. Yo usé la misma receta, pero hice más puré. Si pagué $35 por mis manzanas amarillas, ¿cuánto pagué por las rojas?

Rompecabezas de factores

Rojo Amarillo

El problema se representa con esta proporción:

$$6:14 = c:35$$

Para resolver esta proporción, podemos hacer un rompecabezas de factores con las razones.

$$c \text{ es } 3 \times 5 = 15$$

El rompecabezas de factores se hace con las hileras de la tablas de razones que son × 2 (• 2) y × 5 (• 5) de la razón básica 3:7. Los rompecabezas de factores sirven para resolver problemas difíciles de proporciones.

Comente con su hijo acerca de situaciones de su vida diaria en las que se usen proporciones, tal como hacer el doble de alguna receta de cocina.

Si tiene cualquier pregunta, por favor comuníquese conmigo.

Atentamente,
El maestro de su hijo.

Tabla de razones

	R		A
Bolsas	3	:	7
1	3	:	7
2	6	:	14
3	9	:	21
4	12	:	28
5	15	:	35
6	18	:	42
7	21	:	49
8	24	:	56
9	27	:	63

CA CC

En la Unidad 1 se aplican los siguientes estándares auxiliares, contenidos en los *Estándares estatales comunes de matemáticas con adiciones para California:* **6.RP.1, 6.RP.2, 6.RP.3, 6.RP.3a, 6.RP.3b, 6.EE.6, 6.EE.9, 6.NS.4,** y todos los de prácticas matemáticas.

CA CC Content Standards **6.RP.3, 6.RP.3a**
Mathematical Practices **MP.1, MP.5, MP.6, MP.7**

► Discuss Patterns in the Multiplication Table

Look for patterns in the multiplication tables.

Table 1

•	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54
7	7	14	21	28	35	42	49	56	63
8	8	16	24	32	40	48	56	64	72
9	9	18	27	36	45	54	63	72	81

Table 2

•	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54
7	7	14	21	28	35	42	49	56	63
8	8	16	24	32	40	48	56	64	72
9	9	18	27	36	45	54	63	72	81

► Strategies for Finding Factors

Write the missing factors and the missing product.

1. Table 3

•									
			6		10				
			21		n				

Factor Puzzle

6	10
21	

▶ Solve Factor Puzzles

Write the missing factors and the missing product.

2. Table 4

•								
	6				21			
	16				n			

Factor Puzzle

6	21
16	

3. Table 5

•	2	4	7	1	5	3	6	8	9
3	6	12	21	3	15	9	18	24	27
1	2	4	7	1	5	3	6	8	9
4	8	16	28	4	20	12	24	32	36
2	4	8	14	2	10	6	12	16	18
7	14	28	49	7	35	21	42	56	63
9	18	36	63	9	45	27	54	72	81
5	10	20	35	5	25	15	30	40	45
8	16	32	56	8	40	24	48	64	72
6	12	24	42	6	30	18	36	48	54

Factor Puzzle

	6
35	15

Factor Puzzles and the Multiplication Table

▶ Practice with Factor Puzzles

Solve each Factor Puzzle.

1.

20	15
8	

2.

8	14
	21

3.

	28
24	32

4.

24	
32	36

5.

20	35
	56

6.

	16
35	56

7.

21	24
28	

8.

	10
63	18

9.

35	
15	24

10.

24	28
54	

11.

56	63
	81

12.

54	63
24	

13.

36	63
8	

14.

32	36
	63

15.

	54
56	48

► Make Factor Puzzles

Make your own Factor Puzzles. Exchange with a classmate.

16.
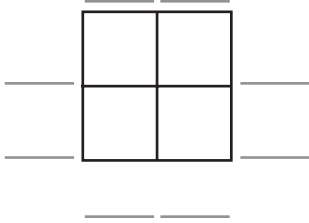
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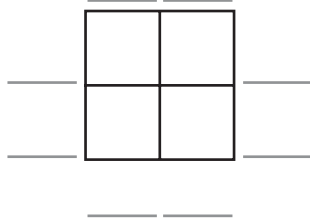
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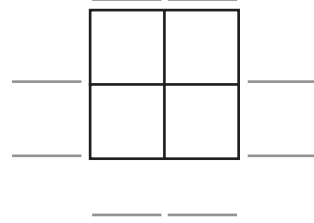
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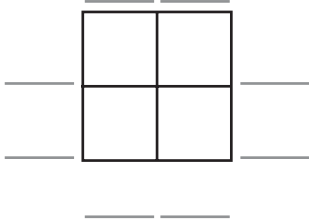
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21.

22.

23.

24.

25.

26.

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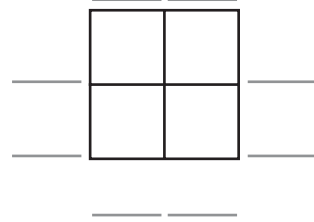
28.

29.

30.

Solving Factor Puzzles

CA CC Content Standards **6.RP.2**, **6.RP.3**, **6.RP.3a**,
6.RP.3b Mathematical Practices **MP.1**, **MP.2**, **MP.3**, **MP.6**,
MP.7, **MP.8**
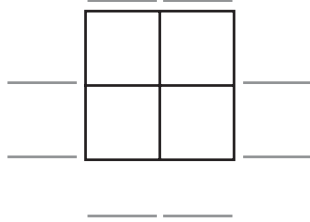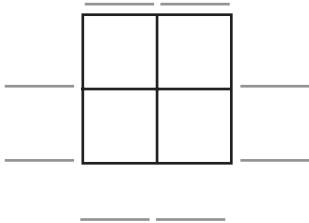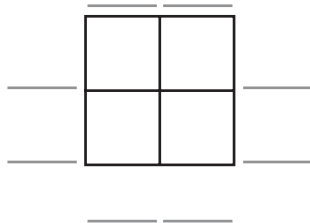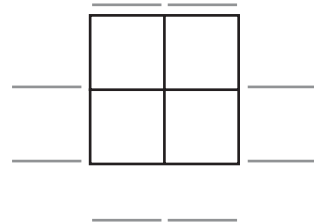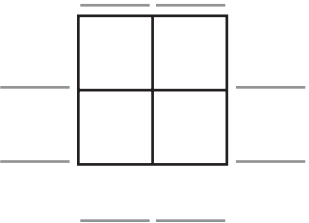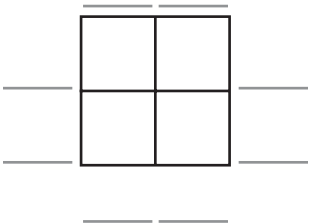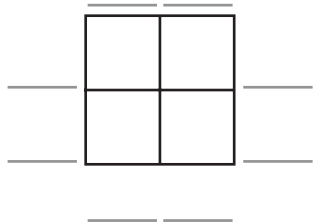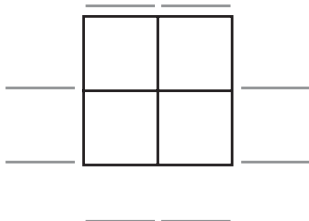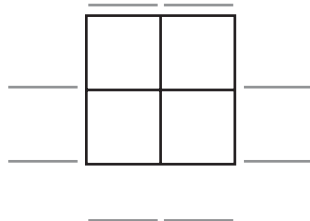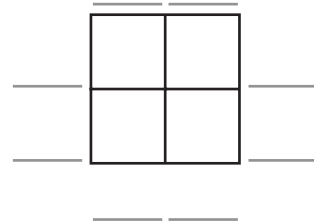
▶ Find the Total So Far

Noreen started to save money. Every day she put three $1 coins
into her duck bank. Write how much money she had each day.

Noreen begins with an empty bank.		Noreen begins with $0 in her bank.
1. On Day 1 Noreen put $3 into her bank.		On Day 1 Noreen had $ _____ in her bank.
2. On Day 2 Noreen put $3 into her bank.		On Day 2 Noreen had $ _____ in her bank.
3. On Day 3 Noreen put $3 into her bank.		On Day 3 Noreen had $ _____ in her bank.
4. On Day 4 Noreen put $3 into her bank.		On Day 4 Noreen had $ _____ in her bank.
5. On Day 5 Noreen put $3 into her bank.		On Day 5 Noreen had $ _____ in her bank.
6. On Day 6 Noreen put $3 into her bank.		On Day 6 Noreen had $ _____ in her bank.

Noreen continues to put $3 into her bank every day.

7. On Day 10, Noreen had _____ in her bank.

8. On Day 12, Noreen had _____ in her bank.

► Complete a Rate Table

This **rate table** shows Noreen's savings.

9. Fill in the rest of the table to show how much money Noreen saved each day and how much her total was each day.

Days	Dollars
1	3
2	6
3	

) + 3

10. What did you write between each row? _____

11. What does the number between each row show?

_____

_____

► Identify Rate Tables

These tables show four different ways Noreen could have saved money. Complete each table. Then decide which tables are rate tables and which are not. Explain why.

12.

Days	Dollars
1	2
2	
3	
4	
5	
6	

) + 2
) + 2
) + 2
) + 2
) + 2

_____

_____

_____

_____

_____

_____

▶ Identify Rate Tables (continued)

13.

Days	Dollars
1	4
2	12
3	18
4	20
5	24
6	28

_____

_____

_____

_____

_____

14.

Days	Dollars
1	7
2	14
3	21
4	28
5	35
6	42

_____

_____

_____

_____

_____

15.

Days	Dollars
1	3
2	5
3	5
4	9
5	11
6	14

_____

_____

_____

_____

_____

▶ Practice Factor Puzzles

Solve each Factor Puzzle.

16.

	32
27	72

17.

10	12
	54

18.

	24
21	28

19.

8	
12	27

20.

24	30
28	

21.

	12
10	8

22.

56	24
35	

23.

	24
28	21

24.

16	56
	21

Make your own Factor Puzzles. Exchange with a classmate.

25.

26.

27.

28.

29.

30.

CA CC Content Standards **6.RP.2**, **6.RP.3**, **6.RP.3a**, **6.RP.3b** Mathematical Practices **MP.2**, **MP.6**, **MP.7**, **MP.8**

▶ Rates and Multiplication

1. Pedro and Pilar collect snails. Each day they add 4 snails to their tank. How many snails do they have after 5 days?

_____

2. We can make a rate table to find the answer for any number of days. Fill in the rest of the rate table. Write the multiplications to the left of the table.

3. Where in the rate table is the answer to Problem 1 and why is it there?

_____

_____

_____

_____

Unit	Product
Days	Snails
1	4
2	8
3	
4	
5	

} +4

▶ Equal-Groups Multiplication and Rates

A rate problem can be thought of as an equal-groups multiplication.
The multiplier is a unit that counts the number of groups.
The **unit rate** is the amount in 1 group.

number of equal groups	•	amount in 1 group	=	total
multiplier	•	unit rate	=	product
number of days	•	4 snails each day	=	total number of snails so far

If the same rate is repeated, the rate is a **constant rate.**
Look for special words that show a constant rate.

The rate is 4 snails **every** day.

The rate is 4 snails **each** day.

The rate is 4 snails **per** day.

► Identify Rate Situations

For each situation, decide whether there is a constant rate.
If *yes*, write the rate and complete the rate table.

4. In the zoo, 7 kangaroos live in each kangaroo house.

Is there a constant rate? _____

_____ _____ per _____ .

Unit	Product
_____	_____
1	
2	
3	

) + ___
) + ___

5. In the last 3 weeks, Ben saw 3 films, then 4 films, and then 3 films.

Is there a constant rate? _____

_____ _____ per _____ .

Unit	Product
_____	_____
1	
2	
3	

) + ___
) + ___

6. Tara made 9 drawings on each page of her sketchbook.

Is there a constant rate? _____

_____ _____ per _____ .

Unit	Product
_____	_____
1	
2	
3	

) + ___
) + ___

7. A bagging machine puts 3 oranges, then 5 oranges, and then 4 oranges in a bag.

Is there a constant rate? _____

_____ _____ per _____ .

Unit	Product
_____	_____
1	
2	
3	

) + ___
) + ___

8. There are 7 days in every week.

Is there a constant rate? _____

_____ _____ per _____ .

Unit	Product
_____	_____
1	
2	
3	

) + ___
) + ___

9. A bagging machine always puts 5 oranges in a bag.

Is there a constant rate? _____

_____ _____ per _____ .

Unit	Product
_____	_____
1	
2	
3	

) + ___
) + ___

Rate Situations and Unit Rate Language

CA CC Content Standards **6.RP.2**, **6.RP.3**, **6.RP.3a**, **6.RP.3b** Mathematical Practices **MP.3**, **MP.6**

► Is a Constant Rate Reasonable?

For each rate situation, fill in the rate information and the rate table. Label the columns in the table.

Discuss what you assume in order for the situation to be a rate situation in the real world.

1. Every day of this week Joanne made 3 of her free throws.

———— ——————— per ——————— .

Unit ———	Product ———
1	

) + ___
) + ___
) + ___

2. Efrain makes 8 sketches on each page of his drawing book.

———— ——————— per ——————— .

Unit ———	Product ———
1	

) + ___
) + ___
) + ___

3. Abby uses 2 cups of flour in each loaf of bread she makes.

———— ——————— per ——————— .

Unit ———	Product ———
1	

) + ___
) + ___
) + ___

4. Eusebio planted 7 tomato vines in each yard he takes care of.

———— ——————— per ——————— .

Unit ———	Product ———
1	

) + ___
) + ___
) + ___

▶ Is It a Rate Table?

Decide whether each table is a rate table. Explain why or why not.

5. _____ _____   6. _____ _____   7. _____ _____   8. _____ _____

1	9
2	18
3	27
4	36
5	45

1	4
2	5
3	9
4	10
5	14

1	11
2	22
3	33
4	44
5	55

1	3
2	5
3	8
4	10
5	13

_____   _____   _____   _____

_____   _____   _____   _____

9. Make up a story about one table and label the table.

▶ Make Scrambled Rate Tables and Math Drawings

For each rate situation, find the unit rate and write it using *per*.
Make a rate table that includes the given information as the first row
in the table. Continue making a **scrambled rate table**.

10. The store sold 5 sacks of oranges
    for $30.

    _____ _____ per _____.

Unit	Product
_____	_____

11. Grandpa's rectangular garden has
    24 pepper plants in the first 4 rows.

    _____ _____ per _____.

Unit	Product
_____	_____

12. Make a math drawing to show
    the first row of your rate table
    for Exercise 10.

13. Make a math drawing to show
    the first row of your rate table
    for Exercise 11.

CA CC Content Standards **6.RP.2**, **6.RP.3**, **6RP.3a**, **6.RP.3b**, **6.EE.6**, **6.EE.9** Mathematical Practices **MP.1**, **MP.3**, **MP.5**, **MP.6**, **MP.7**

▶ Unit Pricing Situations

These three rate tables show the prices of three different kinds of granola. Each has a different unit price. **Fill in the missing values in each table.**

Table 1

1.

Number of Pounds	Cost in Dollars
1	3
2	
3	
	12
5	
6	
	21

Table 2

2.

Number of Pounds	Cost in Dollars
1	6
2	
3	
	24
	30
10	
	600

Table 3

3.

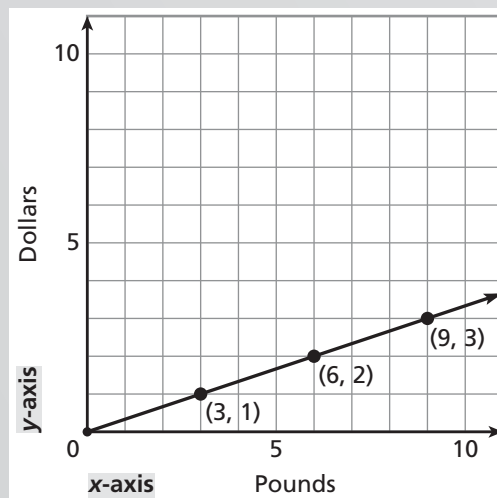| Number of Pounds	Cost in Dollars
1	
8	
	10
6	30
	100
	20
5	

▶ What's the Error?

Dear Math Students,

I made this graph to show the first three rows from Table 1. When I tried to draw the **unit rate triangle**, I realized that I did something wrong.

Can you help me figure out what mistake I made?

Your friend,
Puzzled Penguin

4. Write a response to Puzzled Penguin. Then draw the correct graph in the **coordinate plane** above.

► **Relate Table, Equation, and Graph**

Number of Pounds	Cost in Dollars
1	3
2	6
3	9
4	12
5	15
6	18
7	21

Number of Pounds		Unit Rate	=	Cost in Dollars
p	•	r	=	C
1	•	3	=	3
2	•	3	=	6
3	•	3	=	9
4	•	3	=	12
5	•	3	=	15
6	•	3	=	18
7	•	3	=	21

Number of Pounds		Cost in Dollars
1	• 3	3
2	• 3	6
3	• 3	9
4	• 3	12
5	• 3	15
6	• 3	18
7	• 3	21

The unit rate is circled. Imagine the unit rate written on the vertical rule to be multiplied by the number in the left column to get the number in the right column.
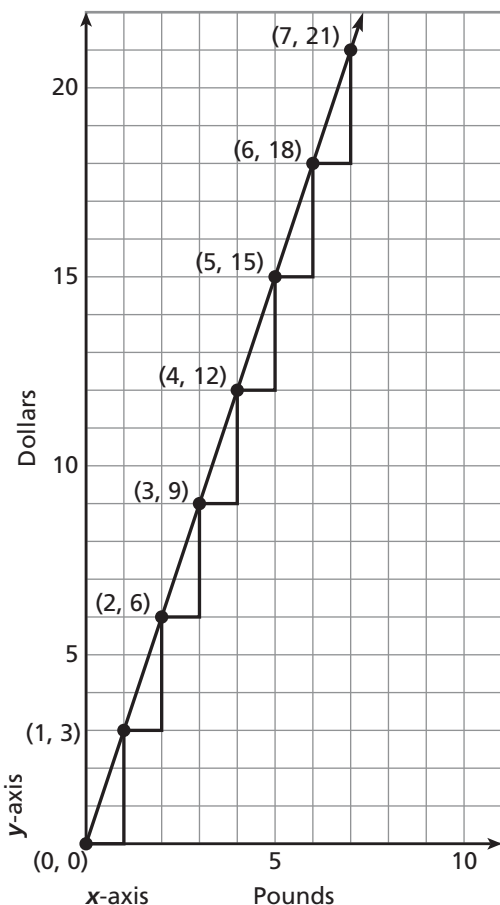
Each point on the graph corresponds to an **ordered pair**. (0, 0) and (1, 3) are ordered pairs.

The first number is the **x-coordinate** and the second number is the **y-coordinate**.

In the ordered pair (1, 3), 1 is the *x*-coordinate and 3 is the *y*-coordinate.

CA CC Content Standards **6.RP.1**, **6.RP.3**, **6.RP.3a**, **6.RP.3b** Mathematical Practices **MP.1**, **MP.3**, **MP.4**, **MP.5**, **MP.6**

▶ Constant Speed Situations

1. Dan ran in the Grade 6 track meet.

Time   Distance

Seconds	Yards
1	
3	15
9	
	10
50	

a. Unit rate: ___ _____ per _____

b. What are the sides of the unit rate triangle for the graph?
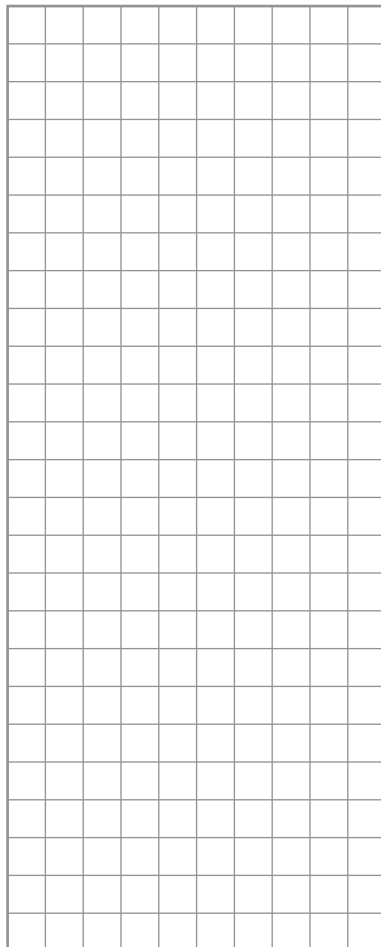
c. Draw the graph.

2. Julie rode her bike in the bike festival.

Time   Distance

Hours	Miles
1	
2	
4	44
	33

a. Unit rate: ___ _____ per _____

b. What are the sides of the unit rate triangle for the graph?

c. Draw the graph.
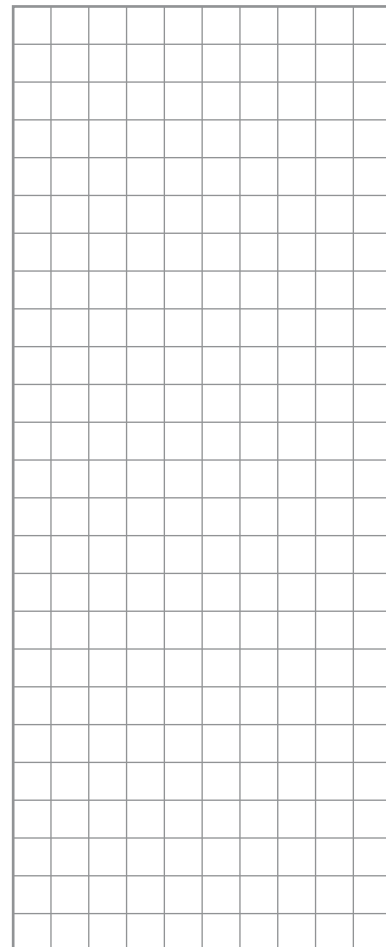
▶ What's the Error?

Dear Math Students,

My friend Gavin also ran in the Grade 6 track meet. His unit rate was 4 yards per second. I made this table of Gavin's times and distances. Gavin says that my table is not correct. Can you help me?

Your friend,
Puzzled Penguin

Time	Distance
Seconds	**Yards**
4	1
8	2
12	3
16	4
32	8
36	9

3. Write a response to Puzzled Penguin.

_____

_____

_____

_____

_____

_____

4. Show the correct table.

▶ Review of Rate Situations

5. Use the graph to fill in the rows of the rate table.
Circle the unit rate in the table. Draw the unit rate triangle in the graph.
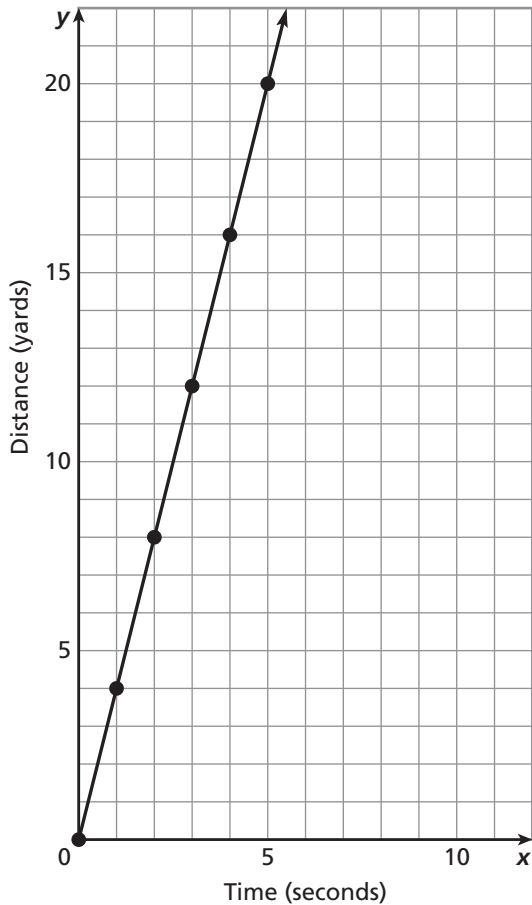
Time	Distance
Seconds	Yards
3	
1	
	8
5	
	16

Solve each rate problem.

6. Ron earns $5 per hour for raking leaves. How much will he earn in 6 hours?

7. Mr. Martin drives at a constant rate of 50 miles per hour for 3 hours. How far does he drive?

8. Margo buys 4 pounds of peaches for $12. What is the unit rate?

9. Karen walks 6 miles in 2 hours. What is her unit rate?

10. **On the Back** Make a rate table for 1 to 5 pounds and draw a graph for the situation in Problem 8.

CA CC Content Standards **6.RP.1**, **6.RP.2**, **6.RP.3**, **6.RP.3a**
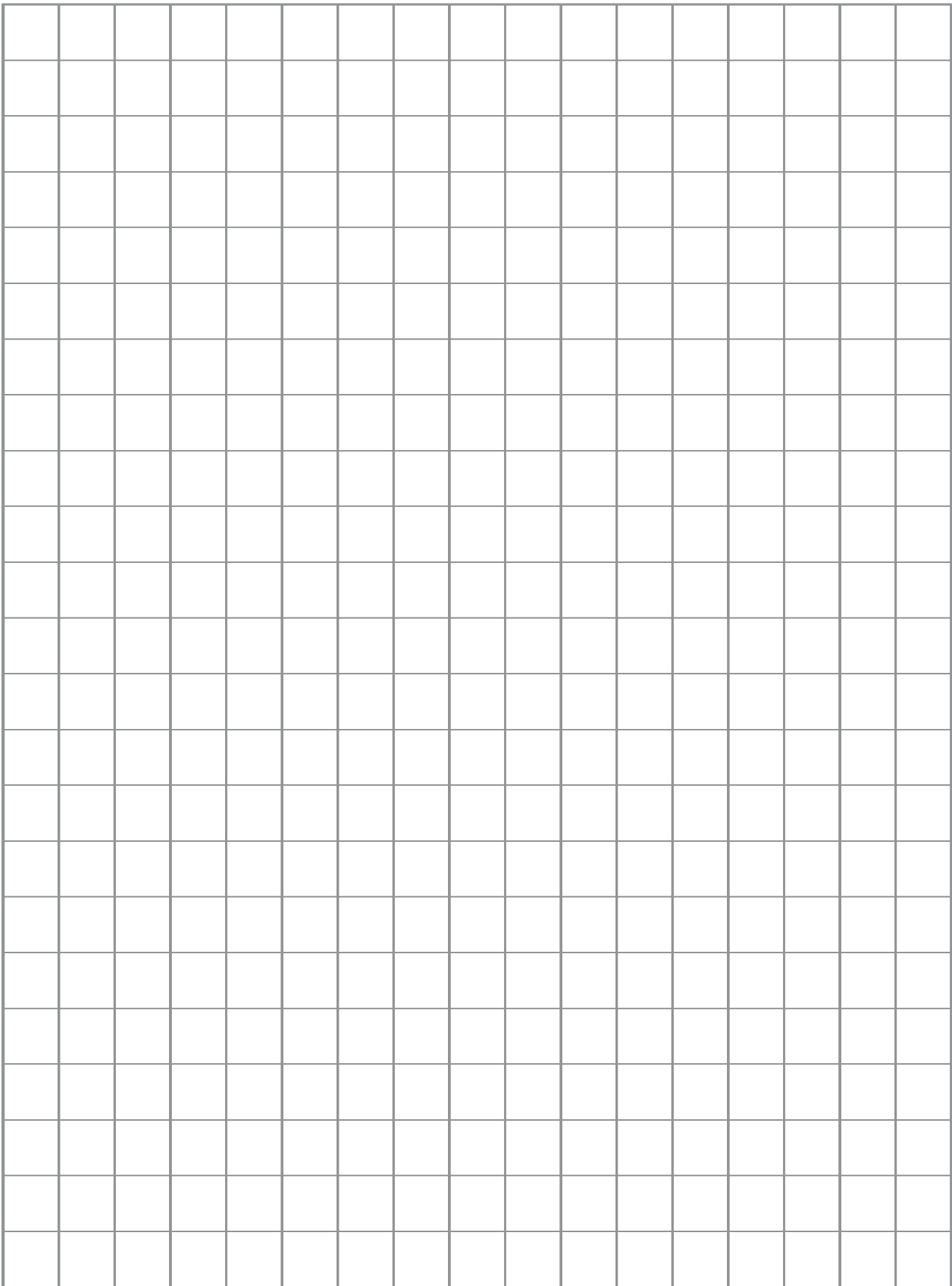Mathematical Practices **MP.3**, **MP.4**, **MP.6**, **MP.7**

▶ Ratio Tables as Linked Rate Tables

Noreen saves $3 a day and Tim saves $5 a day. They start saving on the same day. The **Linked Rate Table** and the **ratio table** show Noreen's and Tim's savings.

Linked Rate Table

Days	Noreen 3	Tim 5
1	3	5
2	6	10
3	9	15
4	12	20
5	15	25
6	18	30
7	21	35

Ratio Table

	Noreen 3	Tim 5	
+3	3	5	+5
+___	6	10	+___
+___	9	15	+___
+___	12	20	+___
+___	15	25	+___
+___	18	30	+___
+___	21	35	+___
+___			+___

1. How are the tables alike? How are they different?

   _____

   _____

   _____

2. What are the numbers circled at the top of each table? _____

3. Fill in the numbers to the left and right of the ratio table to show Noreen's and Tim's constant increases.

Use the tables to answer each question.

4. Noreen has saved $12.

   How much has Tim saved? _____

   On which day was this? _____

5. Tim has saved $35.

   How much has Noreen saved? _____

   On which day was this? _____

6. On what day will Noreen have $30 in her bank? _____

   Why? _____

   How much will Tim have then? _____

▶ Relate Drawings and Ratio Tables

Discuss relationships you see.

• How is the ratio table related to the multiplication table?

• How is the ratio table related to two rate tables?

• How are the constant increases shown in the drawing and in the ratio table?

B	Or
2	3

	B	Or
	2	3
	4	6
	6	9
×4	8	12
	10	15

	B	Or
	2	3
×1	2	3
×2	4	6
×3	6	9
×4	8	12
×5	10	15

B	Or
2	3
2	3
4	6
6	9
8	12
10	15

+2 ... +3

CA CC Content Standards **6.RP.1, 6.RP.3, 6.RP.3a**
Mathematical Practices **MP.2, MP.3, MP.5, MP.6**

▶ Make a Ratio Table

Make a ratio table for each situation. Be sure to label the columns.

1. Noreen makes 2 drawings on each page of her sketchbook. Tim makes 5 drawings on each page of his sketchbook.

2. Two bands march onto the football field. Band 1 marches on in rows of 5 and Band 2 marches on in rows of 7.

3. John can plant 7 tomato vines in the time it takes Joanna to plant 4 tomato vines.

Ratio Table 1	**Ratio Table 2**	**Ratio Table 3**

The linking unit is _____.   The linking unit is _____.   The linking unit is _____.

▶ Ratio Language and Symbols

4. The ratio of Noreen's drawings to Tim's drawings is _____ to _____.

5. a. The ratio of the people in Band 1 to the people in Band 2 is _____ to

   _____ written as 5:7.

   b. If Band 1 has 15 people on the field, Band 2 has _____ people on

   the field. This is _____ rows.

6. a. John and Joanna's tomato vines are in the ratio of _____ to _____.

   b. We write this as _____ : _____.

   c. If John plants 42 tomato vines, Joanna plants _____.

   d. If Joanna plants 8 tomato vines, John plants _____.

▶ Recognize Ratio and Non-Ratio Tables

7. Discuss which are ratio tables and why. Tell ratio stories for each ratio table and tell what the linking unit and the column labels are for each story.

A.

◯	◯
4	7
8	14
12	21
16	28
20	35
24	42
28	49
32	56

B.

◯	◯
3	5
6	12
9	18
12	20
15	24
18	30
21	33
24	42

C.

◯	◯
2	9
4	18
6	27
8	36
10	45
12	54
14	63
16	72

D.

◯	◯
2	3
5	6
7	9
11	12
13	15
16	18
20	21
22	24

▶ Basic Ratios and Equivalent Ratios

A **basic ratio** has the least possible whole numbers.
4:7 is a basic ratio because no whole number (except 1)
divides evenly into 4 and 7.

Equivalent ratios are made up of multiples of the basic ratio row.
Equivalent ratios are written as 8:14 = 20:35 or 8:14 :: 20:35.

8. For the tables that are ratio tables, use the tables to write four pairs of equivalent ratios.

A. _____   _____

_____   _____

B. _____   _____

_____   _____

C. _____   _____

_____   _____

D. _____   _____

_____   _____

CA CC Content Standards **6.RP.1**, **6.RP.3**, **6.RP.3a**
Mathematical Practices **MP.1**, **MP.2**, **MP.4**, **MP.6**, **MP.7**

▶ Proportions and Factor Puzzles

Two equivalent ratios make a **proportion**.
Any two rows from a ratio table make a proportion.
In a proportion problem, one of the four numbers is unknown.
Solving a proportion means finding that unknown number.

Proportion problem:

Grandma made applesauce using the same number of bags
of red apples and bags of yellow apples. Her red apples cost $6
and her yellow apples cost $14. I used her recipe but made
more applesauce. I paid $35 for my yellow apples. How much
did my red apples cost?

The problem can be solved by solving this proportion:

$$6{:}14 = c{:}35$$

To solve the proportion, you need to find the value of c.

1. Fill in the ratio table for the problem.

2. Circle the rows of the ratio table that make up the problem.

3. a. What is the value of c? _____

   b. What is the solution to the problem? _____

You know how to solve Factor Puzzles. It is faster to make a
Factor Puzzle than a whole ratio table.

4. Write the numbers from the proportion problem in the Factor
   Puzzle. Solve the Factor Puzzle.

5. Where in the ratio table are the numbers above and below
   the Factor Puzzle? _____

6. Where in the ratio table are the numbers to the left and
   right of the Factor Puzzle? _____

   Write these two numbers with a multiplication sign to the left
   of the yellow column in the ratio table.

7. Where in the ratio table is your answer? _____

Ratio Table

Bags	R	:	Y
1		:	
2		:	
3		:	
4		:	
5		:	
6		:	
7		:	
8		:	
9		:	

Factor Puzzle

Red Yellow

6	14
	35

▶ Solve Proportion Problems

Use Factor Puzzles to solve these proportion problems. Noreen and Tim both do these activities for the same amount of time but at their own constant rates.

Ratio Table

N   T

8. **a.** Noreen saved $20 while Tim saved $35. When Noreen has saved $24, how much will Tim have saved?

_____

b. Fill in the ratio table. Circle the rows that make the Factor Puzzle and write the multipliers for those rows outside the table.

Factor Puzzles

N   T

N   T

9. While Noreen plants 6 tomato plants, Tim plants 10 tomato plants. When Noreen has planted 21 tomato plants, how many will Tim have planted?

_____ plants

N   T

10. When Noreen had collected 6 stickers, Tim had collected 21 stickers. How many stickers will Noreen have when Tim has 56 stickers?

_____ stickers

11. Noreen did 72 push-ups while Tim did 32 push-ups. When Tim had done 12 push-ups, how many had Noreen done?

_____ push-ups

Seeing Proportions in Ratio Tables

▶ Make Assumptions

Write the assumptions that must be stated to make the problem a proportion problem. Make and label a Factor Puzzle for the problem. Box the unknown and solve the puzzle and the problem.

1. Two bands march in rows onto the football field. When Band A has 15 people on the field, Band B has 6. When Band B has 14 people on the field, how many people will Band A have on the field?

   _____ _____

   _____

   _____

   _____

   _____
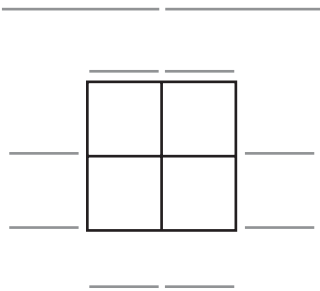
   Problem solution: _____ people

2. Joshua has 32 angelfish for every 12 snails he has. How many snails will he have when he has 72 angelfish?

   _____ _____

   _____

   _____

   _____
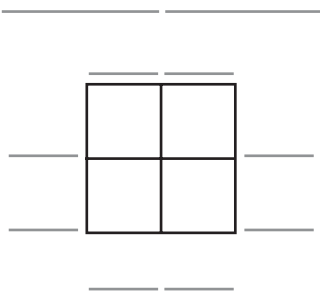
   Problem solution: _____ snails

3. Ann planted 35 rosebushes while Jim planted 14. How many rosebushes had Jim planted when Ann had planted 15?

   _____ _____

   _____

   _____

   _____

   Problem solution: _____ rosebushes

▶ Equivalent Factor Puzzles

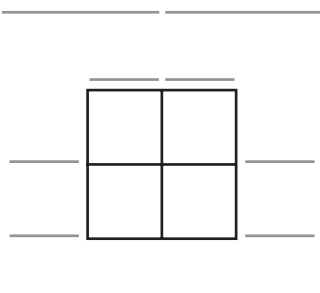**Tell how these Factor Puzzles relate to each other and to the proportion problems.**

For each problem, Ann and Jim plant rosebushes for the same amount of time at their own constant rates.

Factor Puzzle 1

<pre>
      Ann   Jim
       5   2
     ┌─────┬─────┐
  7 │ 35 │ 14 │ 7
     ├─────┼─────┤
  3 │ 15 ││ 6 ││ 3
     └─────┴─────┘
       5   2
</pre>

Proportion Problem 1

Ann planted 35 rosebushes while Jim planted 14. When Ann had planted 15 rosebushes, how many had Jim planted?

Factor Puzzle 2

<pre>
      Jim   Ann
       2   5
     ┌─────┬─────┐
  7 │ 14 │ 35 │ 7
     ├─────┼─────┤
  3 ││ 6 ││ 15 │ 3
     └─────┴─────┘
       2   5
</pre>

Proportion Problem 2

Jim planted 14 rosebushes while Ann planted 35. How many rosebushes had Jim planted when Ann had planted 15?

Factor Puzzle 3

<pre>
      Ann   Jim
       5   2
     ┌─────┬─────┐
  3 │ 15 ││ 6 ││ 3
     ├─────┼─────┤
  7 │ 35 │ 14 │ 7
     └─────┴─────┘
       5   2
</pre>

Proportion Problem 3

When Ann plants 15 rosebushes, how many does Jim plant? Ann plants 35 rosebushes while Jim plants 14.

4. Write the tricky reversed question for each Factor Puzzle.

Factor Puzzle 1: _____

Factor Puzzle 2: _____

Factor Puzzle 3: _____

CA CC Content Standards **6.RP.1, 6.RP.3, 6.RP.3a**
Mathematical Practices **MP.2, MP.3, MP.6**

▶ Numeric Proportions and Factor Puzzles

Show the Factor Puzzle for each proportion.

1. a:32 = 15:40

2. 16:36 = t:63

3. 21:d = 27:18

4. 27:45 :: 21:b

5. 21:f :: 12:20

6. 63:28 :: h:36

▶ Patterns Within Proportions and Factor Puzzles

These proportions have been set up for you.
What patterns do you notice?

7. 2 • 5 : 2 • 12 =
    13 • 5 : 13 • 12

8. 2 • 5 : 2 • 67 =
    39 • 5 : 39 • 67

9. a • c : a • d =
    b • c : b • d

	5	12
2	2 • 5	2 • 12
13	13 • 5	13 • 12

	5	67
2	2 • 5	2 • 67
39	39 • 5	39 • 67

	c	d
a	a • c	a • d
b	b • c	b • d

▶ Differentiate Proportion from Non-Proportion Problems

Circle the number of each problem that is a proportion problem and then solve it with a Factor Puzzle. Circle the basic ratio in each Factor Puzzle. If a problem is not a proportion problem, tell why.

Show your work

10. Peachy Paint Company used 20 cans of blue and 15 cans of yellow paint to make Grasshopper Green paint. They have 8 more cans of blue paint. How many cans of yellow paint do they need to make more Grasshopper Green paint?

_____

_____

11. Mr. Tally's art class uses 2 bags of markers each week. Mr. Petro's art class uses 3 bags of markers one week, 2 bags the next week, and continues this pattern. If Mr. Tally used 14 bags of markers, how many did Mr. Petro use?

_____

_____

12. Jason keeps a constant ratio of minnows to goldfish in his pond. In the summer Jason's pond had 14 minnows for each 6 goldfish. Now it has 27 goldfish. How many minnows does it have now?

_____

_____

13. Tom is 12 years old. He is 8 years older than his sister Sylvia. How old were Tom and Sylvia 3 years ago?

_____

_____

CA CC Content Standards **6.RP.1, 6.RP.3, 6.RP.3a, 6.NS.4** Mathematical Practices **MP.3, MP.6**

▶ Proportions with Basic Ratios

These proportions use a basic ratio. Show a simple solution on your MathBoard.

1. 2:3 :: 10:*a*

$a =$ _____

2. *b*:7 :: 20:28

$b =$ _____

3. What is the basic ratio

for 21:35? _____

▶ Using the Greatest Common Factor

The **greatest common factor** of two whole numbers is the greatest whole number that divides into both numbers without any remainder.

The fastest way to find a basic ratio is to use the greatest common factor. But you can also find the basic ratio by using two or more steps.

4. Discuss Sam's and Asha's strategies for finding the basic ratio for 24:36.

Sam: I put 24 to 36 into a ratio table and divided both numbers by 6. Then I saw that I could divide the numbers in the new ratio 4 to 6 by 2 to get 2 to 3. No whole number (except 1) divides 2 and 3 evenly, so 2 to 3 is the basic ratio.
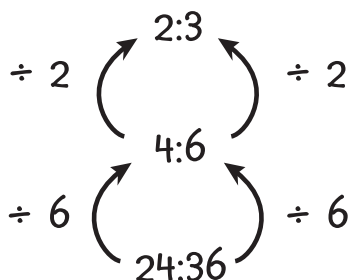
Asha: I know that 24 is 2 times 12 and 36 is 3 times 12, so 12 is a common factor. 24 and 36 could be in the 12s row of the multiplication table. So the basic ratio is 2 to 3.
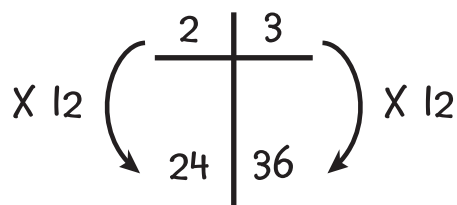
Solve. Show your work.

5. The basic ratio for 60:140 is

_____.

6. The basic ratio for 50:225 is

_____.

► Identify and Solve Basic Ratio Problems

Circle the number of the problem that is not a proportion problem and tell why it is not. Solve each proportion problem.

7. On one street, there are 2 dogs for every 5 cats. If 16 dogs live on the street, how many cats are there?

_____

8. I have 60 blue marbles and 105 red marbles. What is the basic ratio of blue to red?

_____

9. What is the basic ratio of blue to yellow for a paint mixture of 15 cans of blue paint and 21 cans of yellow paint?

_____

10. Six of Susan's cookies weigh the same as 5 of Tara's cookies. How many of Susan's cookies weigh the same as 15 of Tara's cookies?

_____

11. Maggie bought vegetables at the farmers' market. She chose 6 tomatoes and 9 bunches of broccoli. Then she chose 8 carrots. How many heads of lettuce did Maggie buy?

_____

12. Andrew and Barbara begin collecting stamps at the same time. Every week Andrew adds 5 stamps to his collection and Barbara adds 7 to hers. How many stamps will Barbara have collected when Andrew has collected 30 stamps?

_____

1–15
Class Activity

CA CC Content Standards **6.RP.1, 6.RP.2, 6.RP.3, 6.RP.3a** Mathematical Practices **MP.1, MP.2, MP.3, MP.4, MP.5, MP.6**

▶ Math and Hobbies

Model trains are miniature versions of actual trains. Some model trains are very small. The *scale* of a model is a ratio that compares the dimensions of a model train to the dimensions of an actual train.

G Scale 1:25

O Scale 1:48

S Scale 1:64

HO Scale 1:87

N Scale 1:160

Z Scale 1:220
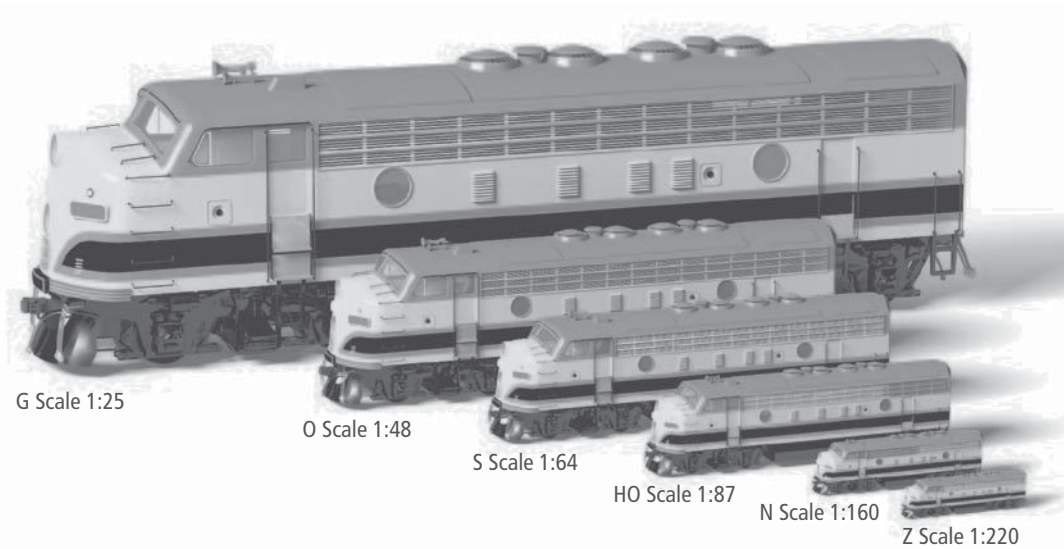
Train hobbyists use scale ratios and proportions to design layouts for their model trains. You can use scale ratios and proportions to find the dimensions of model train cars.

1. The Big Boy was a famous locomotive built during the 1940s. The locomotive was 132 feet long. About how long is an S-scale model of a Big Boy locomotive? (Hint: Use 128 feet instead of 132 feet for your estimate.)

_____

2. The first American-built steam locomotive was about 159 inches long and nicknamed the Tom Thumb because it was so small. About how long is an N-scale model of a Tom Thumb locomotive?

_____

3. An actual container railcar is 76 feet long. About how long is a G-scale model of a container railcar?

_____

▶ Relate Time and Distance

Each year, the New York Botanical Garden displays a garden-scale model train. The buildings are created from plant materials such as, seeds, bark, pods, and stems.

Suppose the model train is traveling at a speed of 2 feet per second.

4. Make a table to show how far the train travels in 5 seconds.

Time	Distance
Seconds	**Feet**
1	
2	
3	
4	
5	

5. Graph the relationship of time and distance.
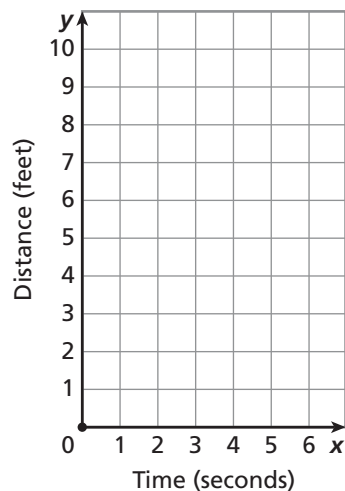
6. The bridge in the photo is about 3 times the height of the train. The buildings are about 4 times the height of the train. If the train cars are 6 inches tall, about how tall is each structure?

1. Is this a proportion problem? Write Yes or No. Explain why or why not.

   Gina puts 3 cups of nuts and 2 cups of raisins in every batch of trail mix that she makes. If she uses 12 cups of nuts, how many cups of raisins will she use?

   _____

2a. Use words and a picture to explain why 2:3 and 8:12 are equivalent ratios.

2b. Select all the ratios below that are equivalent to the ratios in your drawing.

   ○ 4:6

   ○ 6:9

   ○ 9:10

   ○ 12:18

2c. What is another ratio equivalent to 2:3? Explain how you would change your drawing to show that ratio is equivalent.

   Equivalent ratio: _____

Solve each Factor Puzzle.

3.

12	20
21	

4.

	63
40	35

Is each table a rate table? Choose Yes or No. Then in the box below the tables, explain why you answered Yes or No.

5a.   Table 1

Minutes	Miles
1	5
2	10
5	25
7	35
9	45

○ Yes   ○ No

5b.   Table 2

Minutes	Miles
1	5
2	10
5	15
7	20
9	25

○ Yes   ○ No

6a. Complete the rate table.

Pounds	Dollars
1	
	6
3	
4	12
	15

6b. What is the unit rate for the table?

_____

6c. Draw the graph for the rate table.

6d. Show the unit rate triangle.

Use the numbers on the tiles to complete each ratio table.
You may use each number more than once.

2	3	4	5
6	10	11	12
13	14	25	28

7.

Roses	:	Daisies
	:	9
4	:	
8	:	12
	:	15

8.

Apples	:	Oranges
21	:	
6	:	8
	:	16
9	:	

Does the given value for the unknown
solve the proportion? Choose Yes or No.

9a. 4:9 :: 12:a

$a = 3$ ○ Yes   ○ No

$a = 27$ ○ Yes   ○ No

9b. c:10 :: 12:15

$c = 8$ ○ Yes   ○ No

$c = 18$ ○ Yes   ○ No

10. The basic ratio for 12:20 is a:b.

Choose one number from each column to identify the basic ratio.

a	b
○ 2	○ 1
○ 3	○ 3
○ 5	○ 5
○ 8	○ 7
○ 12	○ 9

11. Is the table a ratio table? Write Yes or No, and then explain your answer. If it is a ratio table, write the basic ratio in the top row.

Pencils	Pens
7	6
14	12
21	24
28	38
35	42

12. Match each situation to its unit rate.

9 boxes for $54   6 bags for $42   12 tablets for $24   4 pounds for $12

●   ●   ●   ●

●   ●   ●   ●

1 to 2   1 to 3   1 to 6   1 to 7

13. A baker uses 3 cups of whole wheat flour and 6 cups
of white flour to bake a batch of bread.

Part A

Write a ratio that compares the amount of whole wheat
flour to the amount of white flour.

_____

Part B

The baker bakes a second batch of bread using 4 cups of whole wheat
flour and 12 cups of white flour. Is the ratio of the two flours in this
second batch proportional to the first batch? Explain why or why not.

Part C

A customer wants to buy bread from the batch with the
greater proportion of whole wheat flour. Which batch should
the customer buy from? Support your answer.

14. Tomatoes cost $2 per pound. What is the cost of 6 pounds of tomatoes?

$ _____

15. Melinda rides her bike 18 miles in 2 hours. If she rides at a constant speed, select the answers below that are equivalent ratios to the speed she rides. Select all ratios that are equivalent.

○ 27 miles in 4 hours

○ 9 miles in 1 hour

○ 36 miles in 2 hours

○ 27 miles in 3 hours

○ 36 miles in 4 hours

16. Grow Green garden center sells pots of herbs for 3 for $12. Green Thumb garden center sells the same herbs for 4 for $15.

16a. Which garden center has the lower unit price? Use math facts to support your answer.

16b. Suppose you want to buy 24 pots of herbs. What assumption must you make if you decide to buy them at Green Thumb?

16c. Fill in the missing numbers in each table. Then label each table with the name of the garden center it represents.

_____

Pots	Cost ($)
	30
12	
16	
	75

_____

Pots	Cost ($)
6	
9	
	48
15	

17. Abby saves $3 per week and Luis saves $8. Based on this ratio, complete the chart below to determine how much money Abby and Luis save over time.

Abby ($ per week saved)	Luis ($ per week saved)
3	8
	24
12	
21	
	72

18. Greg mixes 6 cans of black paint with 8 cans of white paint to get a gray paint. How many cans of black paint will he need to mix with 48 cans of white paint to get the same gray color?

Show your work in the space below.

_____ cans

19. Write and solve a proportion word problem for this proportion:

$$6:10 = 21:d$$

Explain why your problem is a proportion word problem.