

Content Overview

## Dear Family,

The goal of this unit of Math Expressions is for your child to become fluent with dividing whole numbers and with comparing, adding, subtracting, multiplying, and dividing fractions and decimals. Below is a summary of the topics in this unit.
Addition and Subtraction of Fractions and Decimals

$$
\frac{2}{3}+\frac{4}{5}=\frac{10}{15}+\frac{12}{15}=\frac{22}{15}=1 \frac{7}{15} \quad \begin{array}{r}
1.40 \\
-0.25 \\
\hline 1.15
\end{array}
$$

## Equivalent Fractions

Multiply the numerator and denominator by the same number.


Divide the numerator and denominator by the same number.


Multiplication of Fractions and Decimals
Multiply numerators and denominators.

$$
\frac{2}{3} \cdot \frac{4}{5}=\frac{2 \cdot 4}{3 \cdot 5}=\frac{8}{15}
$$

Count decimal places in the factors to place the decimal point in the product.

| 0.12 | 0.4 | 0.048 |
| :---: | :---: | :---: |
| 2 | 1 | 3 |
| places | place | places |

## Division of Fractions and Decimals

Multiply by the reciprocal.

$$
\frac{2}{5} \div \frac{3}{7}=\frac{2}{5} \cdot \frac{7}{3}=\frac{14}{15}
$$

Sometimes you can divide numerators and denominators.

$$
\frac{8}{15} \div \frac{2}{3}=\frac{8 \div 2}{15 \div 3}=\frac{4}{5}
$$

If you have any questions or comments, please call or write to me.

## Sincerely, Your child's teacher

## Un vistazo

 general al contenido
## Estimada familia,

El objetivo de esta unidad de Expresiones en matemáticas es que su hijo domine la división de números enteros y que compare, sume, reste, multiplique y divida correctamente fracciones y decimales. Debajo hay un resumen de algunos de los temas de esta unidad.

Suma y resta de fracciones y decimales

$$
\frac{2}{3}+\frac{4}{5}=\frac{10}{15}+\frac{12}{15}=\frac{22}{15}=1 \frac{7}{15} \quad \begin{array}{r}
1.4 Q \\
\hline 1.05
\end{array}
$$

## Fracciones equivalentes

Multiplicar el numerador y el denominador por el mismo número.


$$
\frac{5}{6}=\frac{5 \cdot 2}{6 \cdot 2}=\frac{10}{12}
$$

Dividir el numerador y el denominador entre el mismo número.


## Multiplicación de fracciones y decimales

Multiplicar los numeradores y denominadores.

$$
\frac{2}{3} \cdot \frac{4}{5}=\frac{2 \cdot 4}{3 \cdot 5}=\frac{8}{15}
$$

Contar los lugares decimales en los factores para colocar el punto decimal en la respuesta.

| 0.12 | $0.4=$ | 0.048 |
| :---: | :---: | :---: |
| 2 | 1 | 3 |
| lugares | lugar | lugares |

## División de fracciones y decimales

Multiplicar por el recíproco.

$$
\frac{2}{5} \div \frac{3}{7}=\frac{2}{5} \cdot \frac{7}{3}=\frac{14}{15}
$$

Algunas veces se pueden dividir los numeradores y denominadores.

$$
\frac{8}{15} \div \frac{2}{3}=\frac{8 \div 2}{15 \div 3}=\frac{4}{5}
$$

Dividir el divisor y el dividendo entre el mismo número para convertir el divisor en un número entero.

Aquí se multiplican ambos
por 10.

$$
0 . 2 \longdiv { 0 . 0 8 }
$$

Si tiene preguntas, por favor comuníquese conmigo.

## Atentamente,

El maestro de su hijo

## - Decimal and Whole Number Secret Code Cards



## Decimal and Whole Number Secret Code Cards



## Discuss and Summarize

Fill in the blanks and discuss how the parts of each problem are related.
1 a.

| Place Value |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\times 10$ (larger) |  |  |  |  |  | $\div 10$ (smaller) |
|  |  |  |  |  |  |  |
| Thousands | Hundreds | Tens | ONES | Tenths | Hundredith | Thousandiths |
| 1,000. | 100. | 10. | 1. | 0.1 | 0.01 | 0.001 |
| $\frac{1000}{1}$ | $\frac{100}{1}$ | $\frac{10}{1}$ | $\frac{1}{1}$ | $\frac{1}{10}$ | $\frac{1}{100}$ | $\frac{1}{1000}$ |
| $\frac{\$ 1,000.00}{\left.\frac{60}{6}\right]}$ | $\frac{\$ 100.00}{\frac{135}{2}}$ | \$10.00 | $\$ 1.00$ crioy | $\$ 0.10$ | \$0.01 | \$0.001 |
| 2,000 | $300$ | $60$ |  |  | $0.03$ | $0.002$ |
| $\begin{aligned} & \$ 1,000 \\ & \$ 1,000 \end{aligned}$ | $\begin{aligned} & \$ 100 \\ & \hline \$ 100 \\ & \hline \$ 100 \\ & \hline \end{aligned}$ | $\$ 10$ <br> $\$ 10$ <br> $\$ 10$ <br> $\$ 10$ <br> $\$ 10$ <br> $\$ 10$ | \$1 |  |  | $\nabla_{\Delta}$ |

c. $2,361.632=2,000+$ $\qquad$ $+$ $+$ $\qquad$ $+$ $\qquad$ $+0.002$
2 a.
b.
$\qquad$
$\qquad$
0
. 6
3
b.
c.
d.

|  | 0 | .6 | 0 |
| :---: | :---: | :---: | :---: |
| + | 0 | .0 | 3 |
| + | 0 | .0 | 0 |
|  | 0 | .6 | 3 |

## Discuss Division Meanings

The 49 sixth graders raised $\$ 2,361$ toward their class trip. How much is that for each student?
divisor
quotient remainder

| When $\$ 2,361$ is split 49 ways, each student gets $\$ 40$. Write 4 in the tens place. <br> divisor $\begin{gathered} \downarrow \\ 49 \\ \$ 2,361.00 \\ \uparrow \\ \text { dividend } \end{gathered}$ | Take away the \$1,960 that has been shared out, leaving $\$ 401$ to be shared next. $\begin{gathered} 4 \\ 4 9 \longdiv { \$ 2 , 3 6 1 . 0 0 } \\ -\quad 196 \\ \hline 401 \end{gathered}$ | Each student gets 8 more dollars. Take away the \$392 shared out. $\begin{gathered} 48 \\ \begin{array}{c} \$ 2,361.00 \\ -196 \\ \hline 401 \\ -392 \\ \hline \end{array} \end{gathered}$ | There are 9 dollars left, which is 90 dimes. Each student gets 1 dime, so write 1 in the tenths place. $\begin{gathered} 49 \begin{array}{c} 48.1 \\ \hline \$ 2,361.00 \\ -196 \\ 401 \\ -\quad 392 \\ 90 \end{array} \end{gathered}$ |
| :---: | :---: | :---: | :---: |
| Take away the 49 dimes shared out from the 90 dimes, leaving 41 dimes. $\begin{aligned} & 49 \begin{array}{l} 48.1 \\ \$ 2,361.00 \\ \frac{196}{401} \\ -392 \\ \hline 90 \\ -49 \\ \hline 41 \end{array} \end{aligned}$ | Change the 41 dimes to 410 pennies. $\begin{array}{r} 49.1 \\ 49 \begin{array}{l} \$ 2,361.00 \\ -196 \\ \hline 401 \\ -392 \\ \hline 90 \\ -49 \\ \hline 410 \end{array} \end{array}$ | The multiplier 8, which was used before, works again. Each student gets 8 pennies. $\begin{array}{r} 48.18 \\ \begin{array}{r} \$ 2,361.00 \\ \frac{196}{401} \\ -392 \\ \hline 90 \\ -49 \\ \hline 410 \end{array} \end{array}$ | That makes 392 pennies shared out of the 410 pennies, leaving 18 pennies. $\begin{array}{r} \text { quotient } \rightarrow 48.18 \\ 4 9 \longdiv { \$ 2 , 3 6 1 . 0 0 } \\ -196 \\ \hline 401 \\ \frac{-392}{90} \\ \frac{-49}{410} \\ \begin{array}{r} -392 \\ \rightarrow \quad 18 \end{array} \end{array}$ |

3. Each student gets \$ and there are $\qquad$ cents left over.

## What's the Error?

Dear Math Students.
I solved the problem $5,619 \div 27$ on my math homework. I got 28 R3. I know this can't be right because $27 \times 28$ is approximately equal to $30 \cdot 30$, which is 900. That's not even close to 5,619 ! Will you look at my work and help me figure out what I did wrong?

Your friend,
Puzzled Penguin

4. Write an answer to Puzzled Penguin.

## Pitutic Practice Division

Write each answer as a quotient with a remainder.
5. $2 3 \longdiv { 3 , 2 7 2 }$
6. $1 8 \longdiv { 7 , 3 4 2 }$
7. $1 9 \longdiv { 3 , 8 1 2 }$

## Solve.

8. The engineering club won a $\$ 1,875$ prize in a robot-building contest. The 16 club members want to share the prize equally. How much will each member get? How many cents will be left over?

## What's the Error?

Dear Math Students,

I am really having trouble with my math homework today. I had to solve $3,005 \div 15$. I started, but I got stuck. I know the answer can't be 2, but I don't know what to do next. Can you help me finish the solution?

Your friend,
Puzzled Penguin

9. Write an answer to Puzzled Penguin.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
Ridutio Practice Division
Write each answer as a quotient with a remainder.
10. $1 8 \longdiv { 5 , 4 0 2 }$
11. $1 4 \longdiv { 7 , 0 1 9 }$
12. $3 3 \longdiv { 2 , 1 7 2 }$

## Solve.

13. Twelve friends earned $\$ 4,250$ for painting a house. They want to split the money equally. How much will each friend get? How many cents will be left over?

## Adjust the Multiplier by 1

Discuss each step of Nate's and Caity's solutions. Share how you would do that step.

## Step 1

| Nate | Caity |
| :---: | :---: |
| (90) 6 | (90) 7 |
| 86) 6,123 | 86) 6,123 |
| -516 | -602 |
| Nate Thinks: $7 \bullet 90=630$, which is more than 612, so 7 is too big for the multiplier. I will use 6 . | Caity Thinks: $7 \bullet 90=630$, which is more than 612 , but $6 \bullet 90=540$, which is far from 612. 86 is near the middle between |
| $6 \bullet 6$ ones $=36$. I write the 6 ones under | 80 and 90 , so I think I will use 7. |
| the 2 and the little 3 tens under the 1 , so I remember to add them in. $6 \times 8$ tens $=48$ tens, plus the 3 tens $=51$ tens. | । know $7 \bullet 6$ ones $=42$. I write the 2 and remember the 4 tens to add in. 7 times the 8 tens in 86 is 56 tens, plus the |
| I cross out the 3 because I already added it. So I get 516 . | 4 tens $=60$ tens. So I have 602 , which is really 602 tens because the 7 is in the tens place. |

Step 2

| Nate | Caity |
| :---: | :---: |
| $\begin{gathered} (90) \quad 6 \\ 8 6 \longdiv { 5 _ { 5 } ^ { 5 1 0 1 2 } } \\ -516 \\ \hline \frac{516}{96} \end{gathered}$ <br> Nate Thinks: I need to ungroup to subtract 516 from 612 . I think of 612 as 6 hundreds 1 ten and 2 ones. <br> I ungroup 1 ten and get 10 ones and combine them with the 2 ones to make <br> 12. I need more tens so I ungroup <br> 1 hundred to make 10 tens. Now all the top numbers are greater, so I can subtract. $612-516=96$. I see that 96 is greater than 86 . So the multiplier is 7 , not 6. | $\begin{gathered} (90) \quad 7 \\ 8 6 \longdiv { 6 , 1 2 3 } \\ -602 \\ \hline 10 \end{gathered}$ <br> Caity Writes: 612 - 602 is 10 . <br> The multiplier 7 worked because 10 is less than 86. |

## Adjust the Multiplier by 1 (continued)

## Step 3 (Caity has no step to match Nate's Step 3.)

| Nate |  |
| :---: | :---: |
| $\begin{gathered} (90) \quad 7 \\ 8 6 \longdiv { 5 _ { 5 } ^ { 5 1 0 1 2 0 } } 1 8 3 \\ -602 \\ \hline 10 \end{gathered}$ | Nate Thinks: 7 • 6 ones is 42 , which I write with a little 4 to add in later. <br> $7 \bullet 8$ tens $=56$ tens, plus the 4 tens I already had, equals 60 tens. So I have 602 (which is really 602 tens because the multiplier 7 is in the tens place). $612-602=10$. |

## Step 4

| Nate | Caity |
| :---: | :---: |
| $\begin{array}{r} (90) \quad 71 \\ 86 \begin{array}{\|} \hline 512123 \\ -602 \\ \hline 103 \\ \hline-86 \\ \hline 17 \end{array} \end{array}$ <br> Answer: 71 R17 <br> Nate Thinks: I bring down the 3 to make 103. I can subtract only one group of 86 from 103. <br> I write 1 in the ones place in the quotient. In 103, there are 10 tens, so I regroup 1 ten to make 13 ones. I subtract in each column. $103-86=17$. So, 17 is the remainder. | $\begin{array}{r} (90) \\ 86 \lcm{6,123} \\ -602 \\ \hline 103 \\ -86 \\ \hline 17 \end{array}$ <br> Answer: 71 R17 <br> Caity Thinks: I bring down the 3 to make 103. There is just 1 group of 86 in 103. <br> I write 1 in the quotient. Then I subtract $1-86$ from 103, and the answer is 17. So, the remainder is 17 . |

1. Write an equation to show Nate and Caity how to check their answers.

## Summarize Multiplying by 0.1 and 0.01

Multiplying by 0.1 is the same as dividing by 10 .
$0.1 \cdot 32.4=32.4 \div 10=3.24$
Multiplying a number by 0.1 divides it into 10 parts and takes 1 part. It gives a smaller number. Each digit moves one place right.

You can use this idea to multiply any number of tenths.

## Fill in the blanks.

1. a. To find $0.2 \bullet \$ 32.40$, divide $\$ 32.40$ into 10 parts and take $\qquad$ of those parts.
$0.2 \cdot \$ 32.40=(\$ 32.40 \div 10)$ • $\qquad$
b. To find 6.2 • $\$ 32.40$, divide $\$ 32.40$ into $\qquad$ parts and take $\qquad$ of those parts.
$6.2 \cdot \$ 32.40=(\$ 32.40 \div$ $\qquad$
$\qquad$
Multiplying by 0.01 is the same as dividing by 100 .
$0.01 \cdot 32.4=32.4 \div 100=0.324$
Multiplying a number by 0.01 divides it into 100 parts and takes 1 part. It gives an even smaller number. Each digit moves two places right.

You can use this idea to multiply any number of hundredths.
2. a. To find 0.02 • $\$ 32.40$, divide $\$ 32.40$ into $\qquad$ parts and take $\qquad$ of those parts.
$0.02 \cdot \$ 32.40=(\$ 32.40 \div 100)$ • $\qquad$
b. To find 0.62 • $\$ 32.40$, divide $\$ 32.40$ into $\qquad$ parts and take $\qquad$ of those parts.
$0.62 \cdot \$ 32.40=(\$ 32.40 \div$ $\qquad$
$\qquad$

## Rule About Multiplying Decimals

We can state a rule about multiplying with decimal numbers.

Rule: Ignore the decimal points and multiply. Then place the decimal point so the number of decimal places in the product is equal to the total number of decimal places in the factors.
3. Find each product in two ways: 1) Use the rule

## Example

0.4 • 0.02

Ignore the decimal point and multiply: $4 \cdot 2=8$ Place decimal point so there are three decimal places:
$0.4 \cdot 0.02=0.008$

| 1 | 2 | 3 |
| :---: | :---: | :---: |
| place | places | places | above; 2) Use the method of dividing into 10 or 100 parts and then multiplying by the number of tenths or hundredths.

Show or explain your work. The first one is done for you.
Use the Rule

| a. $0.2 \cdot 0.3$ | 0.06 ; two decimal places | $\underline{(0.3 \div 10) \bullet 2=0.03 \cdot 2=0.06}$ |
| :--- | :--- | :--- |
| b. $0.02 \cdot 0.3$ |  |  |
| c. $0.2 \cdot 3$ |  |  |
| d. $0.02 \cdot 3$ |  |  |
| e. $2 \cdot 0.3$ |  |  |
| f. $2 \cdot 0.03$ |  |  |

4. Explain why the two methods give the same result.

Use either method to solve these problems.
5. $0.04 \cdot 2=$ $\qquad$ 6. $0.4 \cdot 2=$ $\qquad$ 7. $0.3 \cdot 0.3=$ $\qquad$ 8. $3 \cdot 0.3=$ $\qquad$
9. $0.04 \cdot 6=$ $\qquad$ 10. $0.5 \cdot 7=$ $\qquad$ 11. $0.3 \cdot 0.8=$ $\qquad$ 12. $7 \cdot 0.6=$ $\qquad$

## Summarize Dividing by 0.1

Dividing by 0.1 gives a number ten times original number.
$79.6 \div 0.1=10 \cdot 79.6=796$
Dividing by 0.1 is equivalent to multiplying by 10 because there are 10 small parts (tenths) in each whole.

## Discuss and explain.

1. Dividing by 0.1 is the same as multiplying by 10.

Discuss the reason for each numbered step.
$\frac{79.6}{0.1}=\frac{79.6}{0.1} \cdot 1 \underset{(2)}{=} \frac{79.6}{0.1} \cdot \frac{10}{10}=\frac{79.6 \cdot 10}{0.1 \cdot 10}=\frac{796}{1}=796$
2. Multiplying a problem in long-division format by $1=\frac{10}{10}$ gives an equivalent problem with a whole number divisor.
$0 . 1 \longdiv { 7 9 . 6 } \rightarrow 0 . 1 \longdiv { 7 9 . 6 } \quad$ OR
$0 . 1 \longdiv { 7 9 . 6 _ { \wedge } }$

## Complete.

3. The process in Exercise 2 can be used to change any decimal divisor to a whole number. Use the method to rewrite each problem with a whole number divisor.
a. $0 . 4 \longdiv { 2 . 7 }$
b. $7 2 . 5 \longdiv { 0 . 3 9 }$
c. $2 . 7 \longdiv { 9 }$
d. $6 . 3 \longdiv { 5 2 }$
4. What makes Parts c and d of Exercise 3 trickier than the others?

## PATH to

 FLUENCY
## Divide.

5. $0 . 3 \longdiv { 9 }$
6. $0 . 7 \longdiv { 2 . 1 }$
7. $1 . 2 \longdiv { 4 8 0 }$
8. $2 . 2 \longdiv { 0 . 8 8 }$
9. $0 . 5 \longdiv { 1 0 0 }$
10. $3 . 2 \longdiv { 9 6 }$
$1 1 . 0 . 9 \longdiv { 1 0 . 8 }$
11. $0 . 8 \longdiv { 0 . 6 4 }$

## Summarize Dividing by 0.01

Dividing by 0.01 gives a number 100 times the original number.
$79.6 \div 0.01=7,960=100 \cdot 79.6$
Dividing by 0.01 is equivalent to multiplying by 100 because there are 100 small parts (hundredths) in each whole.

Discuss and explain.
13. Dividing by 0.01 is the same as multiplying by 100.

Discuss the reason for each numbered step.

$$
\frac{79.6}{0.01}=\frac{79.6}{0.01} \cdot 1=\frac{79.6}{0.01} \cdot \frac{100}{100}=\frac{79.6 \cdot 100}{0.1 \cdot 100}=\frac{7,960}{1}=7,960
$$

14. Multiplying a problem in long-division format by $1=\frac{100}{100}$ gives an equivalent problem with a whole number divisor.

$$
0 . 0 1 \longdiv { 7 9 . 6 } \rightarrow 0 . 0 1 \longdiv { 7 9 . 6 0 } \quad \text { OR } \quad 0.01 / \sqrt{79.60_{\wedge}}
$$

## Complete.

15. The process in Exercise 14 can be used to change any decimal divisor to a whole number. Use the method to rewrite each problem with a whole number divisor.
a. $0 . 0 4 \longdiv { 2 . 7 }$
b. $7 . 2 5 \longdiv { 0 . 3 9 }$
c. $0 . 2 7 \longdiv { 9 }$
d. $0 . 6 3 \longdiv { 5 2 }$
16. What makes Parts a, c, and d of Exercise 15 tricky?

## Divide.

17. $0 . 0 1 \longdiv { 4 0 }$
18. $0 . 0 3 \longdiv { 0 . 9 }$
19. $0 . 1 2 \longdiv { 4 8 }$
20. $0 . 0 7 \longdiv { 0 . 4 9 }$
21. $3 . 2 1 \longdiv { 6 . 4 2 }$
22. $0 . 2 5 \longdiv { 7 . 5 }$
23. $0 . 1 1 \longdiv { 9 9 0 }$
24. $0 . 0 4 \longdiv { 0 . 1 6 }$

## Greater, Equal, or Less

1. Explore what happens when you multiply 4 by different numbers.

Case 1: Multiply 4 by a number greater than 1.


Is the product less than, greater than, or equal to 4 ?
$\qquad$

Case 2: Multiply 4 by 1.


Is the product less than, greater than, or equal to 4 ?

Case 3: Multiply 4 by a number less than 1.

$$
4 \cdot 0.5=
$$

$\qquad$ 0.5 units ? sq. units

Is the product less than, greater than, or equal to 4 ?
2. Multiply 10 by 1 , a number greater than 1 , and a number less than 1. Tell how each product compares to 10.
$\qquad$
$\qquad$
3. Explore what happens when you divide 3
by different numbers.

Case 1: Divide 3 by a number greater than 1.

$$
3 \div 1.5=
$$

$\qquad$

1.5 units \begin{tabular}{c}
? units <br>

| 3 sq. |
| :---: |
| units | <br>

\hline
\end{tabular}

Is the quotient less than, greater than, or equal to 3 ?

Case 2: Divide 3 by 1.
$3 \div 1=$
1 unit 3 units
sq. units

Is the quotient less than, greater than, or equal to 3 ?

Case 3: Divide 3 by a number less than 1.

$$
3 \div 0.5=
$$

$\qquad$
0.5 units $\begin{gathered}\text { ? units } \\ \text { sq. units }\end{gathered}$

Is the quotient less than, greater than, or equal to 3 ?
$\qquad$
4. Divide 12 by 1 , a number greater than 1 , and a number less than 1. Tell how each quotient compares to 12.
$\qquad$

## Multiply or Divide

For each problem, decide whether you need to multiply or divide. Then solve.
5. A wallet size photo measures 2.5 inches by 3.5 inches. What is the area of a wallet-size photo?
6. Mr. Diaz paid $\$ 5.96$ for 4 pounds of broccoli. What was the price per pound?
7. The Washingtons' backyard has an area of 55.48 square meters. If the length of the yard is 7.6 meters, what is the width?
8. In 1976, gas cost about $\$ 0.36$ per gallon. The gas tank of the car Mr. Renzi drove in that year held 22 gallons. How much did it cost him to fill his gas tank?
9. The area of a king size bed is about 42.2 square feet. A queen size bed is about 0.8 times that size. What is the approximate area of a queen size bed? Round to the nearest tenth of a square foot.
10. A rectangular office has an area of 125 square feet. If the length of the office is 9.5 feet, what is the width? Round to the nearest tenth of a foot.

## Greater or Less

## Solve.

11. Amber earns $\$ 8.50$ per hour bagging groceries. Josie, a cashier, earns 1.3 times as much as Amber.

Does Josie earn more or less than Amber?

How much does Josie earn?
13. Stickers with the school mascot cost \$1.15 each. Manuel bought 9 stickers.

Did Manuel spend more or less than \$9?
$\qquad$

How much did Manuel spend?
$\qquad$
15. Caleb is using thin strips of wood to build a picture frame. One of the strips has a width of 1.2 centimeters and an area of 24.96 square centimeters. Is the length of the strip more or less than 24.96 centimeters?
$\qquad$
What is the length of the strip?

## Which Is Greater?

Solve without doing any calculations.
16. Which is greater, $18 \cdot 76$ or $76 \div 18$ ? How do you know?
$\qquad$
$\qquad$
$\qquad$
$\qquad$
17. Which is greater, $0.37 \bullet 45$ or $45 \div 0.37$ ? How do you know?
$\qquad$
$\qquad$
$\qquad$
$\qquad$
18. Which is greater, $81 \div 12$ or $81 \div 0.12$ ? How do you know?
$\qquad$
$\qquad$
$\qquad$
$\qquad$
19. Which is greater, $23 \cdot 67$ or $0.23 \cdot 67$ ? How do you know?
$\qquad$
$\qquad$
$\qquad$
$\qquad$

## How Many Decimal Places?

Use the fact that $39 \bullet 74=2,886$ to solve each problem.

| 20. $0.39 \cdot 7,400=$ | 21. $0.39 \cdot 740=$ | 22. $0.39 \cdot 74=$ | 23. $39 \cdot 7.4=$ |
| :---: | :---: | :---: | :---: |
| 24. $0 . 7 4 \longdiv { 2 , 8 8 6 }$ | 25. $0 . 7 4 \longdiv { 2 8 8 . 6 }$ | 26. $0 . 7 4 \longdiv { 2 8 . 8 6 }$ | 27. $7 4 \longdiv { 2 8 8 . 6 }$ |
| 28. $3.9 \cdot 740=$ | 29. $3.9 \cdot 74=$ | 30. $3.9 \cdot 7.4=$ | 31. $39 \cdot 0.74=$ |
| 32. $7 . 4 \longdiv { 2 , 8 8 6 }$ | 33. $7 . 4 \longdiv { 2 8 8 . 6 }$ | 34. $7 . 4 \longdiv { 2 8 . 8 6 }$ | 35. $7 4 \longdiv { 2 8 . 8 6 }$ |

32. $7 . 4 \longdiv { 2 , 8 8 6 }$
33. $7 . 4 \longdiv { 2 8 8 . 6 }$
34. $7 . 4 \longdiv { 2 8 . 8 6 }$
35. $7 4 \longdiv { 2 8 . 8 6 }$
$\underset{\substack{\text { Path to } \\ \text { FLUENCY }}}{ }$ Solve.
36. $0.7 \cdot 0.3=$ $\qquad$ 37. $0.06 \cdot 7=$ $\qquad$ 38. $8 \cdot 0.4=$ $\qquad$
37. $0 . 1 2 \longdiv { 4 2 }$
38. $3 . 2 \longdiv { 2 . 4 }$
39. $0 . 0 6 \longdiv { 5 4 }$
40. $0 . 2 7 \longdiv { 1 . 3 5 }$
41. $\begin{array}{r}0.09 \\ \times \quad 52 \\ \hline\end{array}$
42. $\begin{array}{r}8.5 \\ \times \quad 4.2 \\ \hline\end{array}$
43. $\begin{array}{r}7.2 \\ \times \quad 0.25 \\ \hline\end{array}$
44. $\begin{array}{r}18 \\ \times \quad 0.6 \\ \hline\end{array}$
45. $3 . 0 2 \longdiv { 9 0 . 6 }$
46. $7 . 5 \longdiv { 0 . 0 6 }$
47. $0 . 8 \longdiv { 6 . 8 }$
48. $0 . 2 \longdiv { 0 . 9 5 }$

## PATH to

Solve.
51. $0.09 \cdot 0.7=$ $\qquad$ 52. $50 \cdot 0.5=$ $\qquad$ 53. $1.2 \cdot 1.2=$ $\qquad$
55. $0 . 0 7 \longdiv { 2 . 4 5 }$
56. $1 . 2 \longdiv { 0 . 9 }$
57. $0 . 9 \longdiv { 0 . 6 3 }$
61. $\begin{array}{r}7.5 \\ \times \quad 1.9 \\ \hline\end{array}$
65. $1 . 2 \longdiv { 1 . 0 8 }$
63. $0 . 2 \longdiv { 0 . 0 1 }$
64. $0 . 2 4 \longdiv { 3 6 }$
62. $2 . 0 8 \longdiv { 8 . 3 2 }$

## The Meaning of the Remainder

## Solve.

66. Zola has 575 beads. She needs 32 beads to make a necklace. How many necklaces can she make?
67. At a concert, a band earned $\$ 495$ for selling 36 CDs. How much did they earn per CD?
68. Mr. Perelli made 500 ounces of raspberry jam. He filled as many 32 -ounce jars with jam as he could. How much jam did he have left over?

## Comparing Unit Fractions and Decimals

The bar at the top represents a whole watermelon. Below are unit fractions and unit decimals made by dividing the watermelon into equal parts.


The greater the dividing number is, the smaller the unit fraction or decimal will be. The fraction denominator and the decimal place show the dividing number. So, a greater denominator or decimal place means the whole is divided into smaller parts.

1. $4>3$, but
2. $100>10$, but
$0.01 \bigcirc$
0.1
3. $1,000>100$, but
0.001
0.01

## PRUTHOC Adding and Subtracting Unit Fractions and Decimals

The denominator of a fraction tells the number of equal parts the whole is divided into, and, therefore, the number of unit fractions that are made. The numerator tells how denominator
 many of these unit fractions the fraction represents.


Add.
4. $\frac{3}{9}+\frac{2}{9}=$ $\qquad$
5. $\frac{3}{6}+\frac{2}{6}=$ $\qquad$
6. $0.03+0.02=$ $\qquad$

## Subtract.

7. $\frac{5}{7}-\frac{3}{7}=$ $\qquad$ 8. $\frac{5}{7}-\frac{2}{7}=$ $\qquad$
8. $\frac{5}{9}-\frac{3}{9}=$ $\qquad$
9. $\frac{5}{6}-\frac{2}{6}=$
$\qquad$
10. $0.5-0.3=$
11. $0.5-0.2=$
12. $0.05-0.03=$
13. $0.05-0.02=$

To add or subtract fractions with like denominators, add or subtract the numerators.

To add or subtract decimals with like places, add or subtract the numbers in the places.

## Discuss Adding and Subtracting Mixed Numbers

Add mixed numbers by adding whole numbers and adding fractions. You may need to make a new group after adding.

Example $11 \frac{3}{5}+2 \frac{2}{5}$

A. $1 \frac{3}{5}+2 \frac{2}{5}=1+\frac{3}{5}+2+\frac{2}{5}=1+2+\frac{3}{5}+\frac{2}{5}=3+\frac{5}{5}=3+1=4$
B. $1 \frac{3}{5}$

$$
\frac{+2 \frac{2}{5}}{3 \frac{5}{5}}=3+1=4
$$

Subtract mixed numbers by ungrouping a 1 if needed.
Then subtract whole numbers and subtract fractions.
Example $22 \frac{2}{5}-1 \frac{3}{5}$


Suzy's Method

$$
\begin{aligned}
& \frac{3}{5} \text { to } \frac{5}{5} \text { is } \frac{2}{5} \text { plus } \frac{2}{5}=\frac{4}{5} \\
& 2 \frac{2}{5}=1+\frac{5}{5}+\frac{2}{5} \\
& -\frac{1 \frac{3}{5}}{}=\frac{1+\frac{3}{5}}{\frac{2}{5}+\frac{2}{5}}=\frac{4}{5}
\end{aligned}
$$

Ian's Method
${ }^{1} 2 \frac{2}{5}^{\frac{7}{5}}$
$-\frac{1 \frac{3}{5}}{\frac{4}{5}}$

Youngshim's Method

$$
\begin{array}{r}
2+\frac{2}{5}=1+\frac{7}{5} \\
-1+\frac{3}{5}=1+\frac{3}{5} \\
\hline 0+\frac{4}{5}=\frac{4}{5}
\end{array}
$$

## Pautior Practice with Fractions and Decimals

Complete each statement with $<$ or $>$.
15. $\frac{1}{5} \bigcirc \frac{1}{4}$
16. $\frac{1}{8} \bigcirc \frac{1}{9}$
17. $\frac{2}{5} \bigcirc \frac{2}{7}$
18. $0.6 \bigcirc 0.06$
19. $\frac{3}{5} \bigcirc \frac{3}{4}$
20. $\frac{5}{8} \bigcirc \frac{5}{9}$
21. $\frac{8}{9} \bigcirc \frac{8}{10}$
22. $0.08 \bigcirc 0.8$

If the numerators are the same, the fraction with the lesser denominator is greater because its unit fractions are greater.

Complete each statement with $<$ or $>$.
23. $2 \frac{4}{5}$

24. $6 \frac{1}{3} \bigcirc 7 \frac{1}{2}$
25. 2.9
 3.01
26. $4 \frac{3}{7} \bigcirc 4 \frac{6}{7}$

For mixed numbers, if the whole numbers are different, the number with the greater whole number is greater. If the whole numbers are the same, the number with the greater fraction or decimal part is greater.

Complete each statement with $<$ or $>$.
27. $\frac{5}{8} \bigcirc \frac{3}{8}$
28. $\frac{20}{3} \bigcirc \frac{21}{3}$
29.0.4 0.5
30. $0.04 \bigcirc 0.05$

If the denominators are the same, the fraction with the greater numerator is greater because it represents more unit fractions.

Add or subtract.
31. $\frac{3}{5}+\frac{4}{5}=$
34. $\begin{array}{r}6.1 \\ -\quad 2.2 \\ \hline\end{array}$
35. $6 \frac{1}{3}$
$-2 \frac{2}{3}$
36. $\begin{array}{r}14.32 \\ -\quad 9.56 \\ \hline\end{array}$
32. $\frac{7}{8}-\frac{6}{8}=$ $\qquad$

33. $3 \frac{2}{6}+4 \frac{5}{6}=$ $\qquad$
34. $6 \frac{2}{7}$
$-3 \frac{5}{7}$
35. Jacob used $2 \frac{7}{8}$ yards of fabric to make a banner. He added another $\frac{5}{8}$ yard of fabric to decorate the banner. How much fabric did he use?

## What Are Equivalent Fractions?

Each fraction bar shows a fraction equivalent to $\frac{5}{6}$.
$\square$
A. Split each sixth into two equal parts to make 10 twelfths.
$\square$
Numerically: $\frac{5 \cdot 2}{6 \cdot 2}=\frac{10}{12}$. We have more, but smaller, parts.
B. Split each sixth into three equal parts to make 15 eighteenths.

|  | , | I | T | , | \| | \| | , | I | I | , | , | \| |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\frac{15}{18}$ | I | I | I | I | I | I | I | I | I | I | I | I |
|  | I | I | I | I | I | I | I | , | I | , | , | \| |

Numerically: $\frac{5 \cdot 3}{6 \cdot 3}=\frac{15}{18}$. We have more, but smaller, parts.
C. Split each sixth into four equal parts to make 20 twenty-fourths.


Numerically: $\frac{5 \cdot 4}{6 \cdot 4}=\frac{20}{24}$. We have more, but smaller, parts.
You can make equivalent fractions by

- Multiplying to make more, but smaller, parts, as in the examples above.

Dividing to make fewer, but larger, parts.
For example, to simplify $\frac{10}{12}$, form groups of two twelfths to get 5 sixths.


Numerically: $\frac{10 \div 2}{12 \div 2}=\frac{5}{6} \quad$ Think: $\frac{2}{2} \bullet \frac{5}{6}=\frac{10}{12}$

## Seeing Equivalent Fractions in the Multiplication Table

You can use the multiplication table to find equivalent fractions.

| $\cdot$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 2 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 |
| 3 | 3 | $\mathbf{6}$ | 9 | 12 | 15 | 18 | 21 | 24 | 27 |
| 4 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 |
| 5 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 |
| 6 | $\mathbf{6}$ | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 |
| 7 | $\mathbf{7}$ | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 |
| 8 | $\mathbf{8}$ | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 |
| 9 | 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 |

$$
\begin{array}{lllllllll}
\frac{5}{6} & \frac{10}{12} & \frac{15}{18} & \frac{20}{24} & \frac{25}{30} & \frac{30}{36} & \frac{35}{42} & \frac{40}{48} & \frac{45}{54}
\end{array}
$$

Use the multiplication table to write fractions equivalent to the given fraction.

2. $\frac{4}{7}=\ldots={ }_{C}=$

## Finding a Common Denominator

Example $1 \frac{4}{5}+\frac{3}{10}$

## Jo Anne's Solution

I look at the denominators to see what I need to multiply by.

$$
\frac{4}{5}+\frac{3}{10}=\frac{4 \cdot 2}{5 \cdot 2}+\frac{3}{10}=\frac{8}{10}+\frac{3}{10}=\frac{11}{10}=1 \frac{1}{10}
$$

## Mark's Solution

| I see $5 \cdot 2=10$. | So, I multiply 4 and <br> 5 by 2. | I find how many <br> tenths. |
| :--- | :--- | :--- |
| $\frac{4 \cdot}{5 \cdot 2}=\frac{\text { Now I can add. }}{10}$ | $\frac{4 \cdot 2}{5 \cdot 2}=\frac{4}{10}$ | $\frac{4 \cdot 2}{5 \cdot 2}=\frac{8}{10}$ |$\quad$| $\frac{4}{5}+\frac{3}{10}=\frac{8}{10}+\frac{3}{10}$ |
| :--- |

To compare, add, or subtract two fractions when the denominator of one is a factor of the denominator of the other, change the fraction with the lesser denominator to an equivalent fraction that uses the greater denominator.

## Example $20.2+0.03$

## Shauna's Solution

0.2 is two dimes. That is 20 pennies, so $0.2=0.20$.

$$
\begin{array}{r}
0.20 \\
+\quad 0.03 \\
\hline 0.23
\end{array}
$$

I add like places.
To compare, add, or subtract decimals with different numbers of places, put zeros on the end of the decimal with fewer places to make the number of decimal places in the two numbers equal.

Compare, add, or subtract.
$3.0 .07 \bigcirc 0.4$
4. 0.1

0.06
5. $\frac{2}{5} \bigcirc \frac{8}{15}$
6. $\frac{3}{4}$
 $\frac{8}{12}$
7. $\frac{5}{6}-\frac{11}{24}=$ $\qquad$ 8. $0.4-0.08=$ $\qquad$
9. $0.3-0.02=$ $\qquad$ 10. $\frac{4}{5}+\frac{17}{30}=$ $\qquad$

## What's the Error?

## Dear Math Students,

My friend Wendy had a pizza party. After the party, she had $\frac{1}{3}$ of a cheese pizza left. She also had $\frac{1}{6}$ of a pepperoni pizza. I told Wendy that together she has $\frac{2}{9}$ of a whole pizza left. She says that can't be right.
Can you help me figure out my mistake? How much of a whole pizza does Wendy have?

Your friend,

Puzzled Penguin

11. Write an answer to Puzzled Penguin.

## Vocabulary

## Common Denominators for Other Cases

Case 1: Denominators with no common factor except 1

$$
\frac{2}{3}+\frac{5}{4}
$$

Virginia's Solution: I need to find a common denominator. 3 • 4 is 12 . So, if I multiply the numerator and denominator of each fraction

$$
\begin{aligned}
\frac{2 \cdot 4}{3 \cdot 4}+\frac{5 \cdot 3}{4 \cdot 3} & =\frac{8}{12}+\frac{15}{12} \\
& =\frac{23}{12}=1 \frac{11}{12}
\end{aligned}
$$

by the other factor, they'll both have the denominator 12.

Add or subtract.

1. $\frac{4}{5}+\frac{2}{3}=$ $\qquad$
2. $\frac{3}{5}-\frac{1}{6}=$ $\qquad$

Case 2: Denominators with a common factor that is not one of the numbers

$$
\frac{3}{4}+\frac{5}{6}
$$

Roger's Solution: I think of a number that is a multiple of both denominators. 12 is a multiple of 4 and 6 . Then I think of what to multiply the denominator of each fraction by to get 12 . I multiply the numerator and denominator by

$$
\begin{aligned}
\frac{3}{4}+\frac{5}{6} & =\frac{3 \cdot 3}{4 \cdot 3}+\frac{5 \cdot 2}{6 \cdot 2} \\
& =\frac{9}{12}+\frac{10}{12} \\
& =\frac{19}{12}=1 \frac{7}{12}
\end{aligned}
$$

that number.
Serena's Solution: If I can't think of a common multiple, I just use Virginia's method and multiply the numerator and denominator of

$$
\begin{aligned}
\frac{3}{4}+\frac{5}{6} & =\frac{18}{24}+\frac{20}{24} \\
& =\frac{38}{24}=1 \frac{14}{24}=1 \frac{7}{12}
\end{aligned}
$$ each fraction by the other factor.

Add or subtract.
3. $\frac{5}{6}+\frac{3}{8}=$ $\qquad$
4. $\frac{5}{9}-\frac{1}{6}=$ $\qquad$

## Practice With Common Denominators

Complete each statement with $<$ or $>$.
5. $\frac{5}{6} \bigcirc \frac{7}{9}$
6. $\frac{5}{8} \bigcirc \frac{4}{6}$
7. $\frac{5}{7} \bigcirc \frac{3}{4}$
8. $\frac{3}{5} \bigcirc \frac{2}{3}$

Add or subtract.
9. $\frac{5}{6}+\frac{1}{4}=$ $\qquad$
10. $\frac{2}{5}+\frac{4}{7}=$ $\qquad$
11. $3 \frac{4}{5}+6 \frac{2}{3}=$ $\qquad$
12. $4 \frac{1}{2}+2 \frac{5}{9}=$ $\qquad$
13. $6 \frac{7}{10}-5 \frac{7}{8}=$ $\qquad$ 14. $3 \frac{4}{5}-1 \frac{5}{6}=$ $\qquad$
15. $7 \frac{2}{3}-1 \frac{3}{4}=$ $\qquad$
16. $6 \frac{4}{7}-2 \frac{2}{3}=$ $\qquad$
17. A recipe calls for $\frac{1}{3}$ cup of flour. Another recipe calls for $\frac{3}{4}$ cup of flour. How much flour do you need to make both recipes? $\qquad$

## Strategies for Common Denominators

1. The denominators of several pairs of fractions are shown below.
4, 8
4, 3
4, 6
5, 10
5, 7
8, 12
6, 8
6, 12
6, 7

List each pair of denominators beside the case that best describes it.
Think about the strategy you would use to find a common
denominator for the pair.
Case A One denominator is a factor of the other.
Case B The denominators have no common factors other than 1.
Case C The denominators have a common factor that is not one of the numbers and is not 1.

## - Least Common Multiple

2. The least common multiple (LCM) of two numbers is the least number that is a multiple of both numbers. Find the LCM of each pair of numbers.
a. 4,6 $\qquad$ b. 4,10 $\qquad$ c. 6,8 $\qquad$ d. 6,9 $\qquad$
e. 6,10 $\qquad$
f. 8, 10 $\qquad$
g. 8,12 $\qquad$
h. 9, 12 $\qquad$
i. 10,12 $\qquad$
j. 4, 8 $\qquad$
k. 3, 9 $\qquad$
I. 5, 9 $\qquad$
3. Pairs a through i represent which case? What strategy can you use to find the LCM?
4. Pairs j and k represent which case? How do you know the answer you found is the LCM?
5. Pair I represents which case? How do you know the answer you found is the LCM?

The LCM can be one of the denominators (Case A) or the product of the denominators (Case B) or a multiple of both denominators (Case C). We can always use the product of the denominators as a common denominator.

## Solve Real World Problems

Write an equation and use it to solve the problem.
Show your work.
6. In a frog-hopping contest, the frog that hops farthest in two hops wins. On the second hop, your frog hops $\frac{3}{8}$ yard. He hopped a total of $\frac{15}{16}$ yard. How long was his first hop?
7. The planning committee for a class party bought 6.5 liters of juice. After the party, they had 2.75 liters left. How much juice did the class drink at the party?
8. Diana and James are painting a room together. Diana paints $\frac{1}{3}$ of the room and James paints $\frac{1}{4}$ of the room. How much of the room have they painted?
9. A carpenter cuts a piece 0.75 meters long from a board. The remaining board is 1.5 meters long. How long was the board before the carpenter cut it?
10. A teacher brought some colored beads to class. During class, her students used $1 \frac{2}{9}$ bags of beads. The teacher was left with $1 \frac{5}{6}$ bags. How many bags of beads did she bring to class?
11. Pedro wrote a math riddle for his friends.

Guess my number, I'll tell you more:
Subtract me from $6 \frac{5}{8}$, and you'll get four! What is Pedro's number?

## PATH to <br> FLUENCY <br> Mixed Practice

## Add.

$$
\text { 12. } 0.04+0.1=
$$

14. $\frac{3}{7}+\frac{1}{7}=$ $\qquad$
15. $1.2+2.03=$ $\qquad$
16. $2 \frac{3}{10}+3 \frac{1}{4}=$ $\qquad$

## Subtract.

13. $\frac{2}{3}+1 \frac{2}{9}=$ $\qquad$
14. $\frac{1}{5}+\frac{1}{6}=$ $\qquad$
15. $4 \frac{5}{9}+1 \frac{5}{6}=$ $\qquad$
16. $5.2+2.15=$ $\qquad$

$$
\begin{aligned}
& \text { 20. } \frac{2}{3}-\frac{1}{4}= \\
& \text { 22. } 4 \frac{2}{5}-1 \frac{3}{10}= \\
& 24.1 .37-0.4=
\end{aligned}
$$

25. $3 \frac{3}{10}-2 \frac{5}{8}=$ $\qquad$
26. $\frac{8}{11}-\frac{1}{3}=$ $\qquad$
27. $1 \frac{5}{6}-\frac{1}{3}=$ $\qquad$
28. $1.4-0.25=$ $\qquad$
29. $3 \frac{3}{10}-2 \frac{5}{8}=$
30. $5.6-3.04=$ $\qquad$

## Strategies for Comparing

Look for patterns in the fractions and decimals below.

Below are general and special cases for comparing fractions and decimals.

## General Cases

Case 1: Same denominator or number of places Fraction with greater numerator is greater.


Ignore decimal point.
Greater number is greater.
$0.7>0.5$
Case 2: Same numerator or digits
Fraction with lesser denominator is greater.
Decimal with leftmost non-zero digit is greater.
0.3


Case 3: Different denominators
Find equivalent fractions with a common denominator to
 change fractions to Case 1.

## Case 4: Mixed numbers

Number with greater whole number is greater. If whole

$$
5 \frac{1}{8} \bigcirc 2 \frac{9}{10}
$$

numbers are the same, compare
$4.7>4.07$
fractions using Cases 1-3.

## Special Cases for Fractions

Case 5: Denominators are factors of 10 or 100. Compare the decimal equivalents.


Case 6: One fraction $>\frac{1}{2}$. One fraction $<\frac{1}{2}$. Fraction $>\frac{1}{2}$ is greater.
$\frac{5}{8} \bigcirc \frac{3}{7}$

## Use strategies to compare.

28. 0.76

0.67
29. $\frac{2}{3} \bigcirc \frac{7}{12}$
30. $1 \frac{1}{2} \bigcirc 1 \frac{2}{5}$
31. . $\frac{5}{8} \bigcirc \frac{5}{6}$
32. 0.09

0.1
33. $\frac{4}{9} \bigcirc \frac{3}{5}$

## Multiplying by Fractions and Decimals Less than 1

## Example 1

A. $0.1 \cdot 4=0.1(1+1+1+1)=0.1+0.1+0.1+0.1=0.4=4 \div 10$

$$
\frac{1}{10} \cdot 4=\frac{1}{10}(1+1+1+1)=\frac{1}{10}+\frac{1}{10}+\frac{1}{10}+\frac{1}{10}=\frac{4}{10}=4 \div 10
$$

B. $0.2 \bullet 4=2 \bullet 0.1 \bullet 4=2 \bullet 0.4=0.8$ So, $0.2 \bullet 4=(4 \div 10) \cdot 2$.

$$
\frac{2}{10} \cdot 4=2 \cdot \frac{1}{10} \cdot 4=2 \cdot \frac{4}{10}=\frac{8}{10} \quad \text { So, } \frac{2}{10} \cdot 4=(4 \div 10) \cdot 2 .
$$

## Example 2

Remember, any whole number can be written as a fraction that has a denominator of 1 . So, $4=\frac{4}{1}$ and $w=\frac{w}{1}$.
A. $\frac{1}{d} \cdot w=\frac{1 \cdot w}{d \cdot 1}=w \div d=\frac{w}{d}$

$$
\frac{1}{3} \cdot 4=\frac{1 \cdot 4}{3 \cdot 1}=4 \div 3=\frac{4}{3}
$$


$\frac{1}{3}$ of 4 rectangles is $\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=\frac{4}{3}$.
B. $\frac{n}{d} \cdot w=\frac{n \cdot w}{d \cdot 1}=\frac{n \cdot w}{d}$

$$
\frac{2}{3} \cdot 4=\frac{2 \cdot 4}{3 \cdot 1}=8 \div 3=\frac{8}{3}
$$


$\frac{2}{3}$ of 4 rectangles is $\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}=\frac{8}{3}$.
C. $w \cdot \frac{n}{d}=\frac{w \cdot n}{1 \cdot d}=\frac{w \cdot n}{d}$

$$
4 \cdot \frac{2}{3}=\frac{4 \cdot 2}{3}=\frac{8}{3}
$$



4 groups of $\frac{2}{3}$ is $\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}=\frac{8}{3}$.

Multiplying a number $n$ by a fraction less than 1 produces a product less than $n$ because only part of $n$ is being taken.

## Multiplying Fractions

## Example

$\frac{a}{b} \cdot \frac{c}{d}=\frac{a \cdot c}{b \cdot d} \quad \frac{2}{3} \cdot \frac{4}{5}=\frac{2 \cdot 4}{3 \cdot 5}=\frac{8}{15}$
Here are two different ways to model the product of $\frac{2}{3}$ and $\frac{4}{5}$.
Ginny's Model "I use a number line. First I model $\frac{4}{5}$ (red loop), and then I separate each fifth into 3 equal parts, or thirds (green tick marks). This makes 15 equal parts altogether.
Then I take $\frac{2}{3}$ of each fifth (blue loops), which is $\frac{8}{15}$."


Taylor's Model "I use an area model. First, I model $\frac{4}{5}$ by dividing the area of a square into fifths and shading 4 of them red. To find $\frac{2}{3}$ of those 4 fifths, I divide the area into thirds and shade 2 of them blue. The whole area is divided into fifteenths, and there are 8 fifteenths in which the shading overlaps."


1. Explain how the model at the right shows $\frac{7}{3} \bullet \frac{4}{5}$.
$\qquad$
$\qquad$
$\qquad$
$\qquad$

## Multiplying in Different Cases

## Multiply.

2. $\frac{5}{8} \cdot 9=$ $\qquad$
3. $\frac{3}{4} \cdot 6=$ $\qquad$
4. $\frac{2}{5} \cdot 10=$
$\qquad$
5. $12 \cdot \frac{5}{6}=$ $\qquad$
6. $4 \cdot \frac{2}{7}=$ $\qquad$
$7.7 \cdot \frac{4}{9}=$ $\qquad$
7. $\frac{3}{5} \cdot \frac{5}{9}=$ $\qquad$
8. $\frac{5}{6} \cdot \frac{1}{3}=$ $\qquad$
9. $\frac{3}{7} \cdot \frac{1}{8}=$ $\qquad$
10. $\frac{1}{2} \cdot \frac{1}{5}=$ $\qquad$
11. $\frac{3}{8} \cdot \frac{2}{6}=$ $\qquad$
12. $\frac{4}{9} \cdot \frac{5}{7}=$ $\qquad$
13. $2 \frac{3}{5} \cdot 4 \frac{2}{3}=$ $\qquad$
14. $3 \frac{1}{3} \cdot 4 \frac{2}{5}=$ $\qquad$ 16. $1 \frac{2}{7} \cdot 2 \frac{3}{8}=$ $\qquad$
15. Judy has a $\frac{4}{5}$-liter bottle of juice. Fuchang's bottle of juice contains $\frac{2}{3}$ as much as Judy's bottle.
a. Does Fuchang have more or less juice than Judy? How do you know?
ng's bottle (

## What's the Error?

Dear Math Students,
I am going to make turkey burgers for a barbecue. I'll use $\frac{1}{4}$ pound of turkey for each burger. Since I want to make 6 burgers. I'll need to buy $\frac{1}{24}$ pound of turkey. Does that sound reasonable? Or what mistake did I make?

Your friend,
Puzzled Penguin

18. Write a response to Puzzled Penguin.

Dear Math Students,
I am planting a flower garden! The space I have to plant in is $3 \frac{1}{2}$ feet wide and $4 \frac{1}{2}$ feet long. I told my friend that the area of my flower garden is $12 \frac{1}{4}$ square feet. She says I've made a mistake! Can you help me find my mistake, and the correct area?

Your friend,
Puzzled Penguin

19. Write a response to Puzzled Penguin.
$\qquad$
$\qquad$
$\qquad$
$\qquad$

## Divide with Unit Fractions

## Complete the statements.

1. Three friends divide 2 submarine sandwiches equally. What part of a sandwich does each friend receive?
$2 \div 3=\frac{1}{3} \cdot 2=\frac{1}{3}(1+1)=\frac{1}{3}+\frac{1}{3}=$ $\qquad$


Each friend receives $\qquad$ of a sandwich.
2. If the three friends divide 5 sandwiches equally, what part of a sandwich will each friend receive?
$5 \div 3=\frac{1}{3} \cdot 5=\frac{1}{3}(1+1+1+1+1)$
$=\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=$ $\qquad$

Each friend receives $\qquad$ sandwiches.
3. Sasha has $\frac{1}{3}$ of a sandwich. He divides it into 2 equal parts and gives one part to his sister. What part of a sandwich does his sister receive?

$\frac{1}{3} \div 2=\frac{1}{3} \cdot \frac{1}{2}=$ $\qquad$
His sister receives $\qquad$ of a sandwich.
4. If Sasha divided the $\frac{1}{3}$ of a sandwich into 5 equal parts, what part of a sandwich would each part represent?

$\frac{1}{3} \div 5=\frac{1}{3} \cdot \frac{1}{5}=$ $\qquad$

Each part would represent $\qquad$ of a sandwich.

## Divide with Unit Fractions (continued)

5. How many $\frac{1}{3}$ sandwiches are in 2 sandwiches?
$2 \div \frac{1}{3}=2 \cdot 3=$ $\qquad$
There are three $\frac{1}{3}$ sandwiches in each sandwich
 because $1=\frac{3}{3}$.
So, there are $\qquad$ $\frac{1}{3}$ sandwiches altogether.
6. How many $\frac{1}{3}$ sandwiches are in 5 sandwiches?
$5 \div \frac{1}{3}=5 \cdot 3=$ $\qquad$
There are three $\frac{1}{3}$ sandwiches in each sandwich because $1=\frac{3}{3}$.

So, there are $\qquad$ $\frac{1}{3}$ sandwiches altogether.


You just explored three cases of division:
Case 1: If $w$ and $d$ are whole numbers, then
$w \div d=w \cdot \frac{1}{d}$.
Case 2: If $w$ is a whole number and $\frac{1}{d}$ is
a unit fraction, then $\frac{1}{d} \div w=\frac{1}{d} \cdot \frac{1}{w}$.
Case 3: If $w$ is a whole number and $\frac{1}{d}$ is a unit fraction, then $w \div \frac{1}{d}=w \cdot d$.

The reciprocal of a fraction $\frac{a}{b}$ is $\frac{b}{a}$. (So, the reciprocal of whole number $w$, or $\frac{w}{1}$, is $\frac{1}{w}$.)

To divide a unit fraction by a whole number or a whole number by a unit fraction, multiply by the reciprocal of the divisor.

## Divide with Unit Fractions (continued)

Write the reciprocal of the divisor to make each statement true.
7. $3 \div 4=3$ • $\qquad$
8. $10 \div 2=10$ - $\qquad$ 9. $\frac{1}{6} \div 5=\frac{1}{6}$ -
10. $\frac{1}{3} \div 8=\frac{1}{3}$ - $\qquad$
11. $4 \div \frac{1}{9}=4$ -
12. $7 \div \frac{1}{2}=7$ •
$\qquad$

## Divide in Different Cases

Divide. Look at the Fraction Poster for help.
13. $5 \div 7=$ $\qquad$
14. $\frac{1}{4} \div 2=$ $\qquad$
15. $8 \div \frac{1}{2}=$ $\qquad$
16. $\frac{1}{3} \div 6=$ $\qquad$
17. $9 \div 10=$ $\qquad$
18. $7 \div \frac{1}{9}=$ $\qquad$
19. $12 \div \frac{1}{4}=$ $\qquad$
20. $\frac{1}{5} \div 5=$ $\qquad$
$21.6 \div 11=$ $\qquad$
22. $\frac{1}{8} \div 12=$ $\qquad$
23. $3 \div 2=$ $\qquad$
24. $9 \div \frac{1}{5}=$ $\qquad$
25. $7 \div 3=$ $\qquad$
26. $8 \div \frac{1}{6}=$ $\qquad$
27. $\frac{1}{3} \div 9=$ $\qquad$
28. $1 \div 8=$ $\qquad$
29. $\frac{1}{6} \div 3=$ $\qquad$
30. $16 \div 4=$ $\qquad$
31. $\frac{1}{4} \div 11=$ $\qquad$
32. $2 \div \frac{1}{5}=$ $\qquad$
33. $6 \div \frac{1}{2}=$ $\qquad$

## Solve Word Problems

34. Greenie the grasshopper hopped 3 meters in the length of time it took Tiny the ant to crawl $\frac{1}{5}$ meter. Greenie hopped how many times as far as Tiny crawled? Explain your answer.
$\qquad$
$\qquad$
$\qquad$
35. Tiny crawled how many times as far as Greenie hopped? Explain your answer.
$\qquad$
$\qquad$
$\qquad$

## What's the Error?

## Dear Math Students.

The problem below was marked wrong on my math quiz.

$$
\frac{1}{3} \div 4=\frac{3}{1} \cdot 4=\frac{3}{1} \cdot \frac{4}{1}=\frac{12}{1}=12
$$

Can you describe my mistake and explain how I can find the correct quotient? Thank you.

Your friend,
Puzzled Penguin

36. $\qquad$
$\qquad$
$\qquad$

## Is it Multiplication or Division?

Name the operation needed to solve the problem, and predict the size of the result. Then solve.

1. Sujita makes 6 quarts of lemonade to sell at her lemonade stand. Her friend Tess makes $\frac{2}{3}$ as much lemonade. Does Tess make more or less lemonade than Sujita?

Find the amount of lemonade Tess makes. $\qquad$
2. A tractor loader has a capacity of $\frac{1}{7}$ ton, and will be used to move 2 tons of topsoil. Will the loader be filled fewer or more than 2 times?
$\qquad$
$\qquad$
How many times will the loader be filled?
3. An artist can paint 3 decorative boxes per hour. One day, she painted for $\frac{1}{2}$ hour. Did she paint more or fewer than 3 boxes?

Find the number of boxes the artist painted. $\qquad$
4. Anthony rides his bike 2 miles. Stephen rides his bike $\frac{1}{3}$ mile. Does Anthony ride more or less than twice as far as Stephen?
$\qquad$
$\qquad$
Find how many times as far Anthony rides. $\qquad$

## Greater Than or Less Than?

Multiplying by a fraction less than 1 gives a lesser answer. Dividing by a fraction less than 1 gives a greater answer.

## Multiply or divide.

$5.8 \div 10=$ $\qquad$
6. $4 \cdot \frac{1}{2}=$ $\qquad$ 7. $\frac{2}{3} \cdot \frac{1}{6}=$ $\qquad$
8. $\frac{3}{4} \cdot \frac{3}{4}=$ $\qquad$
9. $5 \div \frac{1}{3}=$ $\qquad$
10. $\frac{1}{4} \div 7=$ $\qquad$
11. $\frac{5}{3} \cdot \frac{3}{6}=$ $\qquad$
12. $7 \div 3=$ $\qquad$
13. $10 \div \frac{1}{5}=$ $\qquad$
14. $\frac{5}{8} \cdot \frac{2}{3}=$ $\qquad$
15. $\frac{3}{4} \cdot \frac{2}{6}=$ $\qquad$
16. $\frac{1}{3} \div 10=$ $\qquad$
17. $3 \frac{1}{2} \cdot 5=$ $\qquad$
18. $4 \cdot 1 \frac{2}{3}=$ $\qquad$
19. $5 \div 4=$ $\qquad$
20. $6 \div \frac{1}{9}=$ $\qquad$
21. $3 \frac{5}{8} \cdot 2 \frac{1}{3}=$ $\qquad$
22. $1 \frac{2}{5} \cdot 3 \frac{1}{3}=$ $\qquad$
23. $8 \div 11=$ $\qquad$
24. $\frac{1}{3} \div 6=$ $\qquad$
25. $12 \div \frac{1}{4}=$

## Real World Problems

Solve.
26. For a snack, the members of an after school club ate five-eighths of 24 bagels. How many bagels did the group eat?
27. One bag of seed is used to plant $\frac{1}{4}$ acre of corn. How many bags of seed are needed to plant 10 acres of corn?
28. In a staring contest, suppose you stare at your friend for 2 minutes without blinking. Your friend stares at you without blinking for $\frac{9}{10}$ of that time. How long does your friend stare?
29. A car traveled 8 miles in 8 minutes. In that same length of time, a three-toed sloth traveled $\frac{1}{5}$ mile. How many times the sloth's distance did the car travel?
30. Three-fourths of a gallon of milk remains in a jug. Six friends want to share the milk equally. How much milk does each friend receive?
$\qquad$
31. A batch of raisin muffins requires $1 \frac{1}{4}$ cups of raisins. How many cups of raisins are needed for $2 \frac{1}{2}$ batches of muffins?

## Multiplication and Division Practice

## Multiply or divide.

$\qquad$ 33. $9 \div 11=$ $\qquad$ 34. $2 \cdot 3 \frac{1}{2}=$ $\qquad$
35. $\frac{2}{3} \cdot \frac{6}{7}=$ $\qquad$ 36. $\frac{1}{2} \cdot \frac{5}{8}=$ $\qquad$ 37. $5 \div \frac{1}{5}=$ $\qquad$
38. $\frac{1}{12} \div 6=$ $\qquad$ 39. $\frac{8}{9} \cdot 3=$ $\qquad$ 40. $\frac{2}{3} \cdot 1 \frac{1}{2}=$ $\qquad$
41. $\frac{1}{7} \div 3=$ $\qquad$
42. $3 \div \frac{1}{7}=$ $\qquad$
43. $\frac{4}{5} \cdot \frac{1}{3}=$ $\qquad$
44. $\frac{1}{8} \cdot 2=$ $\qquad$
45. $5 \cdot \frac{1}{3}=$ $\qquad$ 46. $6 \div 5=$ $\qquad$
47. $8 \div \frac{1}{4}=$ $\qquad$
48. $\frac{3}{7} \cdot \frac{1}{12}=$ $\qquad$
49. $1 \frac{1}{3} \cdot 2 \frac{1}{3}=$ $\qquad$
50. $\frac{1}{10} \div 3=$
52. $\frac{6}{7} \cdot \frac{3}{5}=$ $\qquad$
51. $4 \div \frac{1}{3}=$ $\qquad$

## Finding an Unknown Factor

1. On Gino's farm, there is a rectangular wooded area in a cornfield. This wooded section has area $\frac{8}{15}$ square mile and is $\frac{2}{3}$ mile long. How wide is the wooded section?
$\qquad$

## Step 1

Write an equation.

$$
\frac{2}{3} \cdot \frac{?}{?}=\frac{8}{15}
$$

## Step 2

Look at the denominators.

$$
\begin{aligned}
& \frac{2}{3} \cdot \frac{?}{?}=\frac{8}{15} \\
& 3 \cdot 5=15 \\
& \frac{?}{?}=\frac{?}{5}
\end{aligned}
$$



We can write the division $\frac{8}{15} \div \frac{2}{3}=n$.
This is the same as the unknown-factor multiplication
$\frac{2}{3} \cdot n=\frac{8}{15}$.

## Step 3

Look at the numerators.

$$
\begin{aligned}
& \frac{2}{3} \cdot \frac{?}{5}=\frac{8}{15} \\
& 2 \cdot 4=8 \\
& \frac{?}{5}=\frac{4}{5}
\end{aligned}
$$


2. The mugs at a restaurant hold $\frac{2}{3}$ cup of hot chocolate. The restaurant has $\frac{8}{15}$ cup hot chocolate left in its pot. How many servings of $\frac{2}{3}$ cup are in the pot?

Step 1 Write an equation. $\frac{?}{7} \cdot \frac{2}{3}=\frac{8}{15}$


Step 2 Look at the denominators.
Divide each $\frac{1}{3}$ into 5 equal parts to make fifteenths.
$\frac{?}{5} \cdot \frac{2}{3}=\frac{8}{15}$


Step 3 Look at the numerators.
Take 4 fifteenths from each of the 2 thirds to make $\frac{8}{15}$. $\frac{4}{5} \cdot \frac{2}{3}=\frac{8}{15}$


## Relating Multiplication and Division

Find the unknown factor in each equation. Then rewrite the multiplication as a division equation.

| Multiplication Equation | Related Division Equation |
| :--- | :--- |
| 3. $\frac{2}{3} \bullet \frac{8}{15} \div \frac{2}{3}=\frac{8}{15}$ | $\frac{15}{56} \div \frac{5}{7}=\frac{15}{2}$ |
| 4. $\frac{5}{7} \cdot \frac{15}{56}$ |  |
| 5. $\frac{5}{8} \bullet \frac{1}{2}=\frac{20}{72}$ |  |
| 6. $\frac{3}{4} \bullet \frac{1}{2}=\frac{15}{36}$ |  |

Multiplication and division are inverse operations for all whole numbers, decimals, and fractions. One operation undoes the other.

$$
\frac{2}{5} \cdot \frac{1}{5} \div \frac{1}{5}=\frac{2}{5}
$$

## Divide.

7. $\frac{9}{20} \div \frac{3}{5}=$ $\qquad$ 8. $\frac{16}{45} \div \frac{2}{9}=$ $\qquad$ 9. $\frac{6}{45} \div \frac{2}{9}=$
$\qquad$
The products in each multiplication equation are simplified.
Find the unknown factor. Then choose a related division equation to solve.
8. $\frac{2}{5} \cdot \frac{6}{20}=\frac{3}{10}$ Solve $\frac{6}{20} \div \frac{2}{5}=\frac{}{}$ or $\frac{3}{10} \div \frac{2}{5}=$

9. $\frac{3}{4} \cdot \frac{}{}=\frac{15}{24}=\frac{5}{8} \quad$ Solve $\frac{15}{24} \div \frac{3}{4}=\frac{}{}$ or $\frac{5}{8} \div \frac{3}{4}=\frac{}{}$.
10. Shaun can hop for $\frac{8}{15}$ of a block. Shaun can hop $\frac{2}{3}$ of the distance Lisa can hop. How far can Lisa hop?


$$
S=\frac{2}{3} \cdot L \quad L=S \div \frac{2}{3}
$$

## Unsimplify to Divide

$\frac{2}{3} \div \frac{5}{7}=$ ?
We cannot divide the numerator of $\frac{2}{3}$ by 5 or the denominator by 7 .
To be able to divide, we need to unsimplify $\frac{2}{3}$. To unsimplify we rewrite it as an equivalent fraction so the numerators and denominators divide evenly.

Step $1 \quad \frac{2}{3} \div \frac{5}{7}=\left(\frac{2}{3} \bullet \frac{5}{5} \bullet \frac{7}{7}\right) \div \frac{5}{7}$
$\frac{2}{3}$ unsimplified
$5 \div 5=1$
Step $2=\frac{2 \cdot 5 \cdot 7}{3 \cdot 5 \cdot 7} \div \frac{5}{7}$
$7 \div 7=1$
$=\frac{2 \cdot 7}{3 \cdot 5}$
Step $4=\frac{2}{3} \bullet \frac{7}{5}$
2. Complete.

Step


Step $1 \quad \frac{3}{8} \div \frac{2}{5}=\left(\frac{3}{8} \cdot \frac{2}{2} \cdot \frac{5}{5}\right) \div \frac{2}{5}$
$\frac{3}{8}$ unsimplified

$$
\begin{gathered}
2 \div 2= \\
=\frac{3 \cdot 2 \cdot 5}{8 \cdot 2 \cdot 5} \div \frac{2}{5} \\
5 \div 5=
\end{gathered}
$$

$$
=\frac{3 \cdot 5}{8 \cdot 2}
$$

Step 4

3. Complete.

Step 1

Step 2

Step 3

$$
\begin{aligned}
& \frac{4}{9} \div \frac{3}{8}=\overbrace{\underbrace{\frac{4}{9} \cdot \frac{3}{3} \cdot \frac{8}{8}}_{\frac{4}{9} \text { unsimplified }})} \div \frac{3}{8} \\
& 3 \div 3= \\
&=\frac{4 \cdot 3 \cdot 8}{9 \cdot 3 \cdot 8} \div \frac{3}{8} \\
& 8 \div 8=
\end{aligned}
$$

$$
=\frac{4 \cdot 8}{9 \cdot 3}
$$

Step 4


1. How is the number you divide $\frac{2}{3}$ by in the original division problem related to the number you multiply $\frac{2}{3}$ by in the final multiplication problem?
$\qquad$
$\qquad$

Summary: When dividing fractions, unsimplifying the dividend and then dividing numerators and denominators is the same as multiplying by the reciprocal of the divisor. For example, $\frac{2}{3} \div \frac{5}{7}=\frac{2}{3} \cdot \frac{7}{5}$.

## Divide by Either Method

Divide either by dividing numerators and denominators or by multiplying by the reciprocal of the divisor.
4. $\frac{1}{10} \div \frac{2}{3}$ $\qquad$ 5. $\frac{15}{12} \div \frac{5}{6}$ $\qquad$
6. $\frac{2}{3} \div \frac{3}{4}$ $\qquad$ 7. $2 \frac{1}{4} \div \frac{3}{4}$ $\qquad$
8. $\frac{15}{16} \div \frac{5}{4}$ $\qquad$ 9. $2 \frac{1}{5} \div \frac{2}{3}$ $\qquad$
10. $\frac{2}{7} \div \frac{3}{4}$ $\qquad$ 11. $\frac{4}{10} \div \frac{4}{5}$ $\qquad$
12. $\frac{5}{6} \div \frac{4}{5}$ $\qquad$ 13. $1 \frac{3}{8} \div \frac{1}{2}$ $\qquad$
14. $\frac{18}{24} \div \frac{9}{12}$ $\qquad$ 15. $2 \frac{1}{4} \div 1 \frac{1}{2}$ $\qquad$
16. Choose a problem from Exercises $4-15$ that you solved by dividing numerators and dividing denominators. Solve it again, but this time multiply by the reciprocal of the divisor. Do you get the same answer?

The following division methods give the same answer:
Method 1: Dividing numerators and dividing denominators.
Method 2: Multiplying by the reciprocal of the divisor.

## Divide by Multiplying by the Reciprocal

You can solve any division problem involving two fractions or a fraction and a whole number by multiplying by the reciprocal of the divisor.

You know that you can think of a division problem like this:

$$
\text { product } \div \text { known factor }=\text { unknown factor }
$$

To divide you multiply by the reciprocal of the known factor.

Think: "Flip the factor and multiply."
Study the examples and discuss them with your class.

## Example 1

$$
\begin{aligned}
& \text { fraction } \div \text { fraction } \\
& \frac{5}{6} \div \frac{2}{3}=\frac{5}{2_{2}} \cdot \frac{z^{1}}{2}=\frac{5}{4} \text { or } 1 \frac{1}{4}
\end{aligned}
$$

## Example 3

unit fraction $\div$ whole number

$$
\frac{1}{4} \div 8=\frac{1}{4} \div \frac{8}{1}=\frac{1}{4} \cdot \frac{1}{8}=\frac{1}{32}
$$

## Example 5

fraction $\div$ whole number

$$
\frac{3}{4} \div 5=\frac{3}{4} \div \frac{5}{1}=\frac{3}{4} \cdot \frac{1}{5}=\frac{3}{20}
$$

## Example 2

whole number $\div$ unit fraction

$$
8 \div \frac{1}{4}=\frac{8}{1} \div \frac{1}{4}=\frac{8}{1} \cdot \frac{4}{1}=32
$$

## Example 4

whole number $\div$ fraction

$$
5 \div \frac{3}{4}=\frac{5}{1} \div \frac{3}{4}=\frac{5}{1} \cdot \frac{4}{3}=\frac{20}{3} \text { or } 6 \frac{2}{3}
$$

## Example 6

fraction $\div$ unit fraction
$\frac{2}{3} \div \frac{1}{12}=\frac{2}{\neq B} \cdot \frac{12^{4}}{1}=8$

## Example 7

$$
\begin{aligned}
& \text { unit fraction } \div \text { fraction } \\
& \frac{1}{12} \div \frac{2}{3}=\frac{1}{4 \sqrt{2}} \bullet \frac{x^{1}}{2}=\frac{1}{8}
\end{aligned}
$$

## Practice Dividing Fractions

Divide using any method.

1. $\frac{3}{4} \div \frac{5}{2}$ $\qquad$ 2. $\frac{3}{4} \div 5$ $\qquad$
2. $\frac{8}{25} \div \frac{4}{5}$
3. $\frac{3}{4} \div \frac{1}{5}$ $\qquad$
4. $\frac{7}{8} \div \frac{5}{8}$
5. $\frac{1}{21} \div \frac{1}{7}$ $\qquad$
6. $3 \div \frac{1}{7}$ $\qquad$ 8. $3 \div \frac{4}{7}$ $\qquad$

## Solve.

## Show your work.

9. Marilla's dress had $1 \frac{5}{6}$ yards of blue lace around it. Her sister Ana's dress had $\frac{3}{4}$ yard of purple lace around it. Marilla's lace is how many times as long as Ana's lace?
10. Ms. Padilla divided $\frac{3}{4}$ liter of apricot juice into 5 glasses for her family. How much did each person get?
11. The sixth grade garden at Sunnyside School is $\frac{4}{3}$ yards long and has an area of $\frac{8}{9}$ square yard. How wide is the garden?
$\qquad$
12. Bronwyn's prize-winning yellow zucchini is $1 \frac{1}{6}$ feet long. This is $\frac{3}{4}$ the length of her prize-winning green zucchini. How long is the green zucchini?
13. A pot contains $5 \frac{1}{3}$ cups of oatmeal. How many $\frac{3}{4}$-cup servings is this? How much will be left over?

## Predict, Solve, and Check

Will each product or quotient below be greater or less than the first fraction? Circle your prediction, and then find the actual product or quotient to check your prediction.

1. $\frac{3}{4} \bullet \frac{6}{5}$ Predict: $>\frac{3}{4}$ or $<\frac{3}{4}$
2. $\frac{2}{3} \bullet \frac{3}{5}$ Predict: $>\frac{2}{3}$ or $<\frac{2}{3}$
$\frac{3}{4} \cdot \frac{6}{5}=$ $\qquad$ $\frac{2}{3} \cdot \frac{3}{5}=$ $\qquad$
3. $\frac{9}{10} \div \frac{6}{5}$ Predict: $>\frac{9}{10}$ or $<\frac{9}{10}$
4. $\frac{3}{10} \div \frac{2}{5}$ Predict: $>\frac{3}{10}$ or $<\frac{3}{10}$
$\frac{9}{10} \div \frac{6}{5}=$ $\qquad$ $\frac{3}{10} \div \frac{2}{5}=$ $\qquad$

Solve.
5. Four friends discovered $\frac{3}{4}$ of a pizza in the refrigerator and ate $\frac{2}{3}$ of it for a snack. What fraction did they eat?
a. What fraction is $\frac{3}{4}$ being multiplied by? $\qquad$
b. Is the fraction in Part a greater or less than 1?
c. Will the product be greater or less than $\frac{3}{4}$ ? $\qquad$
d. What fraction of the pizza did the friends eat? $\qquad$
6. A baker placed $4 \frac{2}{3}$ cups of bread flour into small plastic containers that each had a capacity of $\frac{2}{3}$ cup. How many containers did the baker fill?
a. What fraction is $4 \frac{2}{3}$ being divided by? $\qquad$
b. Is the fraction in Part a greater or less than 1? $\qquad$
c. Will the quotient be greater or less than $4 \frac{2}{3}$ ? $\qquad$
d. How many containers did the baker fill? $\qquad$

## Real World Problems

Decide if you need to multiply or divide. Then solve.
7. Todd used $\frac{2}{3}$ cup of oatmeal for a muffin recipe. To make a different recipe, he needs only $\frac{3}{4}$ of that amount. What amount of oatmeal does the different recipe require?
8. Amelia is using a wheelbarrow to move $1 \frac{1}{2}$ cubic yards of topsoil. The capacity of the wheelbarrow is $\frac{1}{6}$ cubic yard. How many wheelbarrow loads will be needed to move the soil?
9. A rectangular field measures $\frac{3}{10}$ mile by $\frac{1}{6}$ mile. In square miles, what is the area of the field?
10. A pitcher contains $\frac{7}{16}$ gallon of lemonade. How many cups, each having a $\frac{1}{12}$-gallon capacity, can be filled from the jug?
11. Carlos jogs the same route each day. The route is $3 \frac{1}{4}$ miles long, and he stops to rest every $\frac{7}{8}$ mile. How many times does Carlos stop to rest during his jog?
12. Last night Julian needed $\frac{5}{12}$ hour to complete his homework. Aaliyah needed only $\frac{4}{5}$ as long. How many minutes did it take Aaliyah to complete her homework?

## Fraction Mixed Review

## Solve.

13. The table at the right uses fractions and mixed numbers to represent the weights and costs of peaches that can be purchased at a roadside fruit stand.

Complete the table.
14. A triangle has a base measure of $5 \frac{1}{2}$ centimeters and a height of $2 \frac{1}{2}$ centimeters. What is the area of the triangle? Use a mixed number in simplest form in your answer.
$\qquad$
15. Quinn knows that the area of a triangle is $\frac{1}{18}$ square yard and its base measures $\frac{1}{9}$ yard. What is the height of the triangle?
$\qquad$

Simplify before multiplying.

Show your work.

| Weight <br> (pounds) | Cost <br> (dollars) |
| :---: | :---: |
| $\frac{3}{4}$ |  |
| 1 | $3 \frac{3}{5}$ |
|  | $4 \frac{1}{2}$ |
| $1 \frac{5}{8}$ |  |

16. $\frac{14}{8} \cdot \frac{32}{42}=$ $\qquad$
17. $\frac{44}{36} \cdot \frac{72}{33}=$ $\qquad$
18. $\frac{16}{24} \cdot \frac{15}{28}=$ $\qquad$
19. $\frac{32}{18} \cdot \frac{21}{20} \cdot \frac{15}{32} \cdot \frac{8}{49}=$ $\qquad$

## What's the Error?

Dear Math Students,
To find the product at the right, I followed the steps below.

$$
2 \frac{1}{3} \cdot 1 \frac{3}{7}
$$

Step 1: Multiply the whole numbers. $2 \cdot 1=2$
Step 2: Multiply the fractions. $\frac{1}{\nmid 1} \bullet \frac{\dot{B}}{7}=\frac{1}{7}$
Step 3: Add the results from Step 1 and Step $2.2+\frac{1}{7}=2 \frac{1}{7}$
I learned that $2 \frac{1}{7}$ was not the correct product. Please explain my mistake, and how I can find the correct product.

Your friend,
Puzzled Penguin
21. $\qquad$
$\qquad$
$\qquad$

Dear Math Students,
To find the quotient at the right, I followed the steps below.

$$
3 \frac{3}{4} \div 1 \frac{1}{3}
$$

Step 1: Change the mixed numbers to fractions. $3 \frac{3}{4}=\frac{15}{4}$ and $1 \frac{1}{3}=\frac{4}{3}$
Step 2: Multiply the fractions. $\frac{\frac{5}{4}}{4} \cdot \frac{\frac{1}{8}}{\neq 1}=\frac{5}{1}=5$
I was told that 5 is not the correct quotient. Please explain my mistake, and how I can find the correct quotient.

Your friend,
Puzzled Penguin

22. $\qquad$
$\qquad$
$\qquad$

## Review Operations with Fractions and Decimals

To add, subtract, or compare fractions and decimals, use $1=\frac{D}{D}$ to make equivalent fractions or decimals with the same denominator.

$$
\begin{array}{lc}
\frac{2}{3}+\frac{4}{5}=\frac{5}{5} \cdot \frac{2}{3}+\frac{3}{3} \cdot \frac{4}{5}=\frac{10}{15}+\frac{12}{15}=\frac{22}{15} & 0.2+0.04=0.20+0.04=0.24 \\
\frac{2}{10} \cdot \frac{10}{10}=\frac{20}{100} \\
\frac{4}{5}-\frac{2}{3}=\frac{3}{3} \cdot \frac{4}{5}-\frac{5}{5} \cdot \frac{2}{3}=\frac{12}{15}-\frac{10}{15}=\frac{2}{15} & 0.2-0.04=0.20-0.04=0.16
\end{array}
$$

To multiply fractions, multiply the numerators and multiply the denominators.
$\frac{2}{3} \cdot \frac{4}{5}=\frac{8}{15}$

To multiply decimals, multiply the numbers and count decimal places in the factors to place the decimal point in the product.

$$
\begin{aligned}
& 0.2 \cdot 0.4=0.08 \\
& \frac{2}{10} \cdot \frac{4}{10}=\frac{8}{100}
\end{aligned}
$$

To divide fractions, divide the numerators and divide the denominators or multiply by the reciprocal.
$\frac{8}{15} \div \frac{2}{3}=\frac{8 \div 2}{15 \div 3}=\frac{4}{5}$
$\frac{8}{15} \div \frac{2}{3}=\frac{8}{15} \cdot \frac{3}{2}=\frac{24}{30}$ or $\frac{4}{5}$

To divide decimals, unsimplify using $1=\frac{D}{D}$ to make the divisor a whole number, or move the decimal points the same number of places.

$$
0 . 0 8 \div 0 . 2 = \frac { 0 . 0 8 } { 0 . 2 } \cdot \frac { 1 0 } { 1 0 } = \frac { 0 . 8 } { 2 } = 0 . 4 \quad 0 . 2 \longdiv { ( 0 . 0 8 }
$$

## Simplify.

1. $\frac{3}{4}+\frac{5}{8}$
2. $\frac{5}{6} \cdot \frac{2}{3}$ $\qquad$ 3. $3 \frac{7}{8}-\frac{1}{2}$
3. $7.6-1.81$ $\qquad$ 5. $2.29+0.8$ $\qquad$ 6. $5.2 \cdot 1.5$
4. $1 \frac{2}{5}+\frac{7}{8}$ $\qquad$
5. $0.27 \div 0.6$
6. $\frac{1}{3}+6 \frac{7}{10}$

## Choose the Operation

## Write and solve an equation.

10. Hala can ride her bike $7 \frac{1}{2}$ miles in an hour. How far will she ride in 3 hours? How far will she ride in $\frac{2}{3}$ of an hour?
11. Jonny can throw a baseball $30 \frac{1}{3}$ yards. Joey can throw a baseball $33 \frac{1}{2}$ yards. How much farther can Joey throw?
12. Eryn's rabbit eats $\frac{5}{16}$ pound of food each day. Eryn buys food in 5 -pound bags. How many days of food are in a bag?
$\qquad$
13. Marcus has soccer practice for 9 hours each week. Luis spends $\frac{5}{6}$ as much time practicing soccer. How much time each week does Luis spend practicing?
14. In a long jump competition, Stacey jumped 10 feet, which was 0.75 foot farther than Reggie jumped. How far did Reggie jump?
15. Evan bought $\frac{3}{8}$ pound of sunflower seeds and $\frac{3}{16}$ pound of thistle seeds for his bird feeder. How many pounds of seeds did he buy?
16. Casandra's fish bowl holds 0.9 gallon. The bowl is $\frac{3}{4}$ full. How much water is in the bowl?
17. Ty practices trumpet for $1 \frac{2}{3}$ hours every day. He plays scales every $\frac{1}{3}$ hour. How often during a practice does Ty play scales?

## Math and the Stock Market

Many people save for retirement by making investments. One example of an investment is to buy shares of stock in a company. Companies use the money they receive from the sale of stock for operating expenses. Most companies have operating expenses of millions or billions of dollars.

When you buy shares of stock in a company, you become a shareholder, which is a part-owner of the company. Shares of stock are bought and sold on a stock exchange like the one shown at the right.

The following terms and definitions are related to buying and selling shares of stock.

Dividend A portion of a company's profits that are paid to shareholders. Dividends are often paid quarterly.

Sales The number of shares sold that day.
High The highest price per share that day.
Low The lowest price per share that day.
Last The price per share for the last sale of the day.

Change The increase or decrease from the price at the end of the previous day.

Stock prices can be given as fractions or decimals.

| Name | Dividend | Sales | High | Low | Last | Change |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Alphabet <br> Foods | - | 72,500 | $\$ 6.44$ | $\$ 6.39$ | $\$ 6.44$ | $+\$ 0.02$ |
| Colossal <br> Corp. | $\$ 0.30$ | 440,000 | $14 \frac{5}{8}$ | $14 \frac{1}{4}$ | $14 \frac{1}{2}$ | $\frac{-1}{8}$ |

## Buying Stocks

The table below shows the cost of one share of the stock of two different companies.

| Company Name | Monday <br> Price | Tuesday <br> Price | Wednesday <br> Price | Thursday <br> Price | Friday <br> Price |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Exploration Inc. | $8 \frac{1}{4}$ | $8 \frac{1}{8}$ | $8 \frac{5}{8}$ | $8 \frac{1}{2}$ | $8 \frac{3}{4}$ |
| Miles of Styles | $\$ 12.04$ | $\$ 12.03$ | $\$ 12.05$ | $\$ 12.02$ | $\$ 11.98$ |

1. Suppose you purchase 125 shares of each company's stock on Monday. Write an equation that represents the cost (c) of each purchase in dollars. Then solve each equation.
$\qquad$
$\qquad$
2. Find the change in price for a share of each company's stock during the times shown below. Write + to indicate an increase in price and write - to indicate a decrease.
a. from Monday to Tuesday
b. from Tuesday to Wednesday
c. from Wednesday to Thursday
d. from Thursday to Friday
3. Suppose you sell your 125 shares on Friday. Calculate the amount of money you should receive for each stock. Did you earn a profit, or lose money?
4. Suppose you had purchased 12,500 shares of each company's stock instead of 125 . How would your profit or loss for each stock change? Use a calculator to help decide.
$\qquad$
$\qquad$

## Select all that apply.

1. $2 \longdiv { 6 2 9 }$
2. $2 5 \longdiv { 2 , 5 1 5 }$
(A) 213
(A) 55
(B) 314 R 1
(B) 95.6
(C) 314.5
(C) 100.6
(D) 345
(D) 100 R 15
3. Is the problem below a multiplication or a division problem?

Explain your answer.
The rectangular floor of a playpen has an area of 14 square feet.
One measure of the floor is $3 \frac{1}{2}$ feet. What is the other measure?
$\square$
4. Select True or False for each statement.
4a. $0.9 \div 7.5=0.12$TrueFalse
4b. $\quad 63.2 \div 8=7.8$TrueFalse
4c. $\quad 72.8 \div 5.2=1.4$TrueFalse

$$
4 d . \quad 7 \div 0.35=20
$$True

False
$\qquad$
5. Without doing any computation, predict if the quotient $73 \div 0.42$ will be greater than or less than the product $0.42 \bullet 73$. Explain your reasoning in terms of the size of 0.42 .
$\square$
Use the numbers and the decimal point (if needed) to write the product. Each tile may be used more than once.
0
6

6. $\begin{array}{r}0.4 \\ \times 0.3 \\ \hline\end{array}$
7. $\begin{array}{r}0.06 \\ \times 12 \\ \hline\end{array}$
$\qquad$
8. The fraction pairs in the table are not equivalent. Find the LCM for each fraction pair. Rewrite each pair as an equivalent
fraction pair using the LCM as the denominator.

| Fraction Pair | $\frac{2}{7}, \frac{1}{3}$ | $\frac{5}{12}, \frac{3}{4}$ | $\frac{7}{10}, \frac{4}{15}$ |
| :--- | :--- | :--- | :--- |
| LCM |  |  |  |
| Equivalent Fraction Pair |  |  |  |

Multiply. Write the product.
9. $\quad 5.8$
2.5
$\times 2$
10. $\quad 2.9$
$\begin{array}{r}0.81 \\ \hline\end{array}$

Product: $\qquad$ Product: $\qquad$

Find the sum or difference. Select one number from each column to make the equation true.

12. $\frac{7}{9}+\frac{5}{6}=$ $\qquad$

| whole <br> number | fraction |
| :---: | :---: |
| $\circ 0$ | $\bigcirc \frac{5}{8}$ |
| $\bigcirc 1$ | $\circ \frac{4}{5}$ |
| $\bigcirc 2$ | $\bigcirc \frac{1}{4}$ |
| $\bigcirc 3$ | $\bigcirc \frac{3}{4}$ |


| whole <br> number | fraction |
| :---: | :---: |
| 0 | $\frac{3}{7}$ |
| 1 | $\frac{7}{9}$ |
| 2 | $\frac{11}{13}$ |
| 3 | $\frac{11}{18}$ |

Write a number in the box to make the equation true.
13. $3 \frac{2}{5}-\square=1 \frac{1}{2}$
14.

15. George calculated a sum of $\frac{15}{24}$.

Which fractions are equivalent to $\frac{15}{24}$ ? Choose all that apply.
(A) $\frac{10}{16}$
(D) $\frac{5}{8}$
(B) $\frac{5}{6}$
(E) $\frac{3}{8}$
(C) $\frac{7}{12}$
(F) $\frac{30}{48}$
16. Read the division problem below. Then select True or False for each statement.

$$
\frac{3}{4} \div 7=\square
$$

16a. To find the quotient, first find a common denominator.TrueFalse

16b. The quotient will be less than $\frac{3}{4}$.

- TrueFalse

16c. You will need to simplify the quotient.

- TrueFalse

Draw a line to match each missing numerator or denominator to its value. You will not use all of the choices.

18. $\frac{1}{4} \cdot \frac{\square}{\square \cdot}=\frac{3}{20}$


Choose the expressions that make the equation true.
Select all that apply.
19. $\frac{8}{15} \div \frac{4}{5}=$
(A) $\frac{1}{3}$
(B) $\frac{2}{3}$
(C) $\frac{20}{60}$
(C) $\frac{26}{24}$
(D) $\frac{40}{60}$
(D) $1 \frac{1}{12}$
21. Seth wrote an expression that simplifies to 0.36 .

Choose Yes or No to indicate if the expression has a value of 0.36 .
21a. $36 \cdot 0.01$
$\bigcirc$ Yes

- No
21b. $3.6 \cdot 0.1$
Yes
○ No
21c. $3.6 \div 0.1$
Yes
O No
21d. $0.036 \div 0.01$
$\bigcirc$ Yes
- No

22. Darnell has two scraps of cloth. The first scrap is 3.75 cm in length. The second scrap is $\frac{1}{5}$ as long as the length of the first.

## Part A

What is the total length of the scraps? Show your work.
$\square$

## Part B

Answer the following question without making a calculation.
Explain how you know the answer.
If Darnell's second scrap was $\frac{1}{3}$ as long as the first, would the total length of the scraps be greater than or less than the answer you found in Part A?
$\square$
23. Lydia has $8 \frac{1}{2}$ cups of juice to serve.

## Part A

If each serving is $\frac{3}{4}$-cup, how many full servings can she make?
What amount of juice, if any, will be left over? Show your work.


## Part B

If she needs 15 servings, how much more juice does she need?
(A) $2 \frac{1}{4}$ cups
(B) 3 cups
(C) $2 \frac{3}{4}$ cups
(D) 4 cups

## Part C

Lydia decides to make each serving of juice $\frac{5}{8}$ cup. Without calculating, predict if she can make more full servings or fewer full servings using the $8 \frac{1}{2}$ cups of juice. Explain your choice. Then confirm your prediction.

Predict:

Confirm:
24. Dana's dog Skippy weighs 20.4 pounds. In comparison, her cat Bruiser weighs 0.75 as much as Skippy.

24a. Does Bruiser weigh more or less than Skippy? Explain.
$\square$
24b. How much does Bruiser weigh?

